

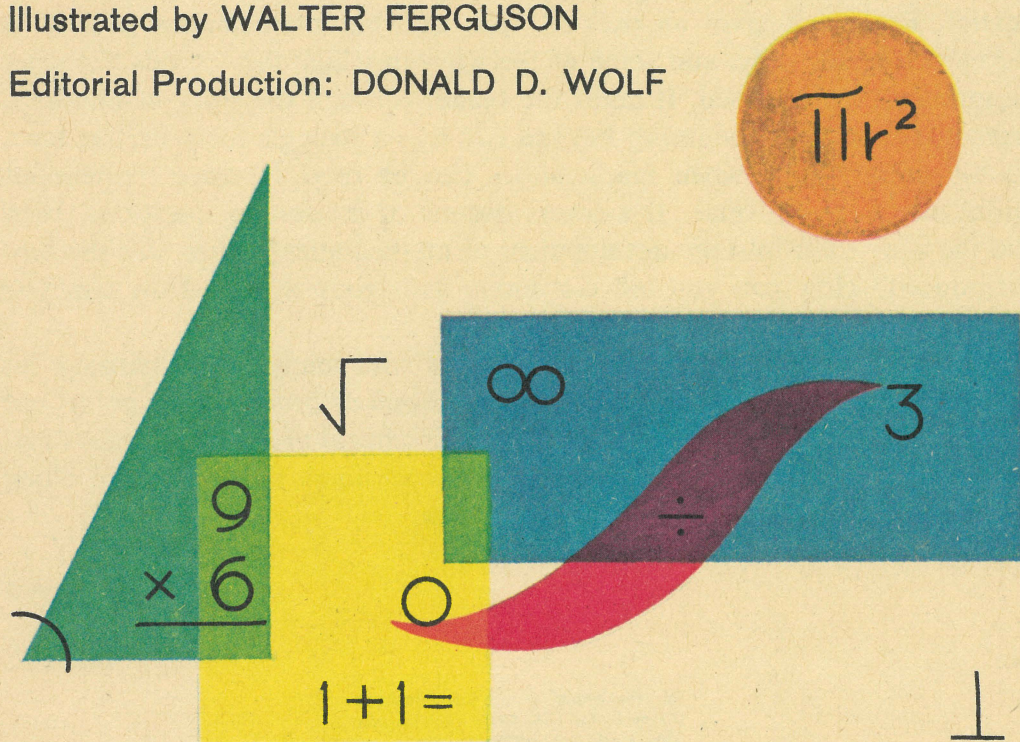
THE HOW AND WHY WONDER BOOK OF MATHEMATICS

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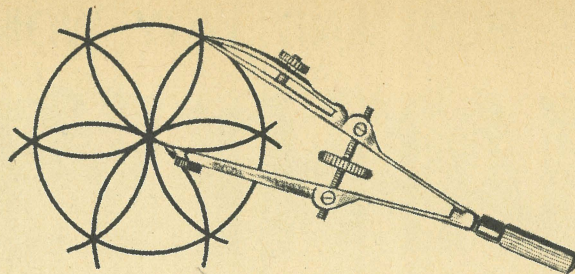
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Introduction

If you wanted to find a *googol*, where would you look? In a zoo? Through a telescope? In a deep well? No, you would look in a mathematics book. Mathematics is not only a science in itself, but it is a very important scientific tool as well.

The How and Why Wonder Book of Mathematics is not intended to give the details of arithmetic, algebra, geometry and other branches of mathematics. Rather, it gives an over-all view of what mathematics is, how it developed historically, and some of the ways in which the various branches of mathematics are used. It takes the reader through the story of numbers, from the time when counting beyond two was a struggle for primitive men, to the present time when mathematics can be used to solve tremendous problems of the universe. It answers dozens of interesting questions. How did the stars enter into the development of mathematics? When was the zero introduced? How can you tell directions with your watch? How can you break codes?

Mathematics is a growing, changing tool with many new directions for mankind's use. *The How and Why Wonder Book of Mathematics* points out some of these new directions. It will certainly appeal to young scientists and mathematicians. It will find good use both at home and at school along with the other *How and Why Wonder Books*.

Paul E. Blackwood

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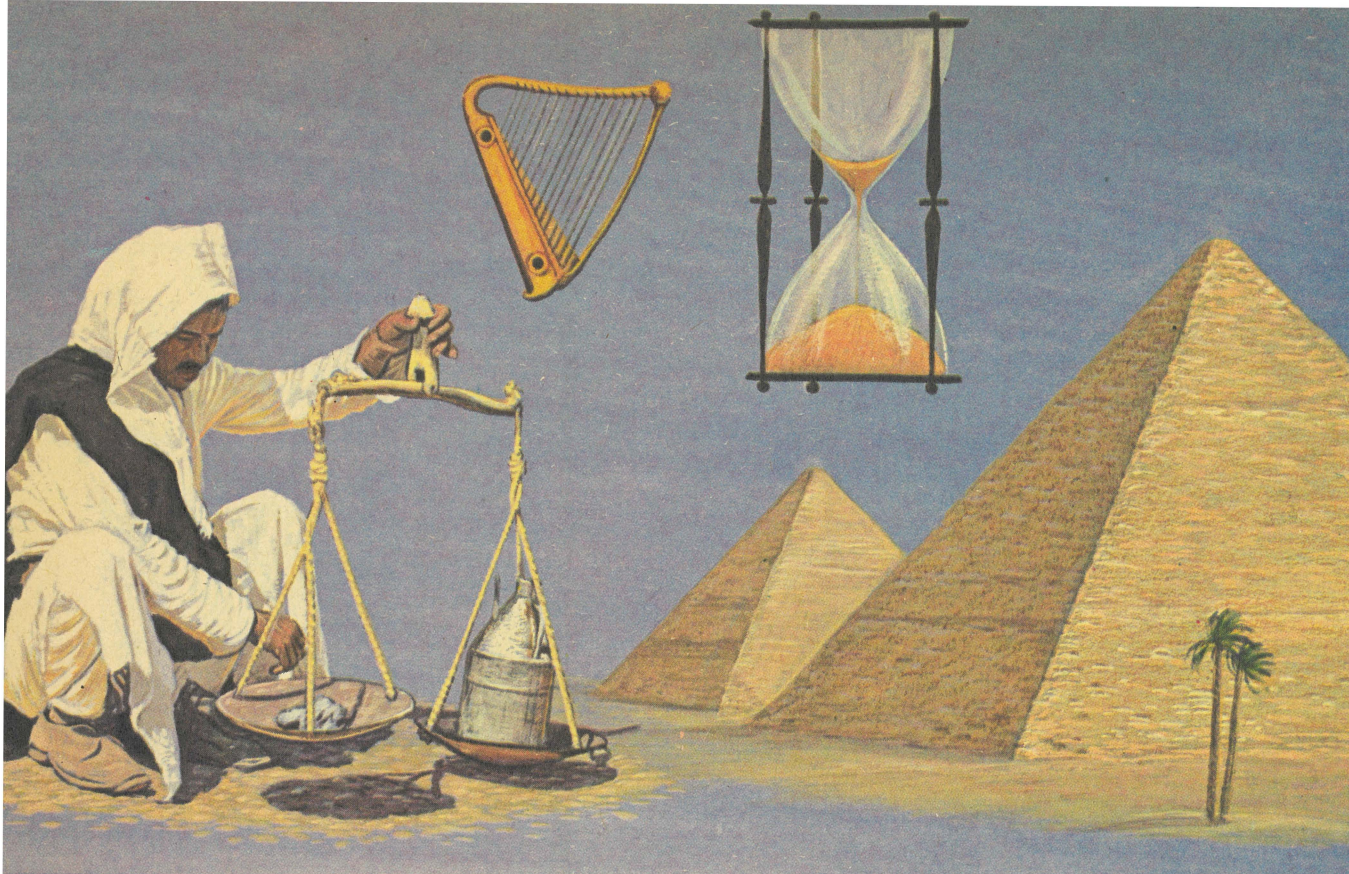
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The Language of Mathematics

When you opened this book, you started on a trip through the world of mathematics. In all its long history, mathematics has never been so exciting as it is today. Revolutionary discoveries and remarkable changes are being made more rapidly than ever before.

How do mathematicians communicate?

You would miss much of the enjoyment of traveling through any new land unless you knew enough of the language to understand what was going on. Communicating mathematical ideas was once a problem even among mathematicians, but they solved this by developing a special language. The next few pages will tell you the meanings of the signs, symbols and words which you need to know to enjoy this book fully.

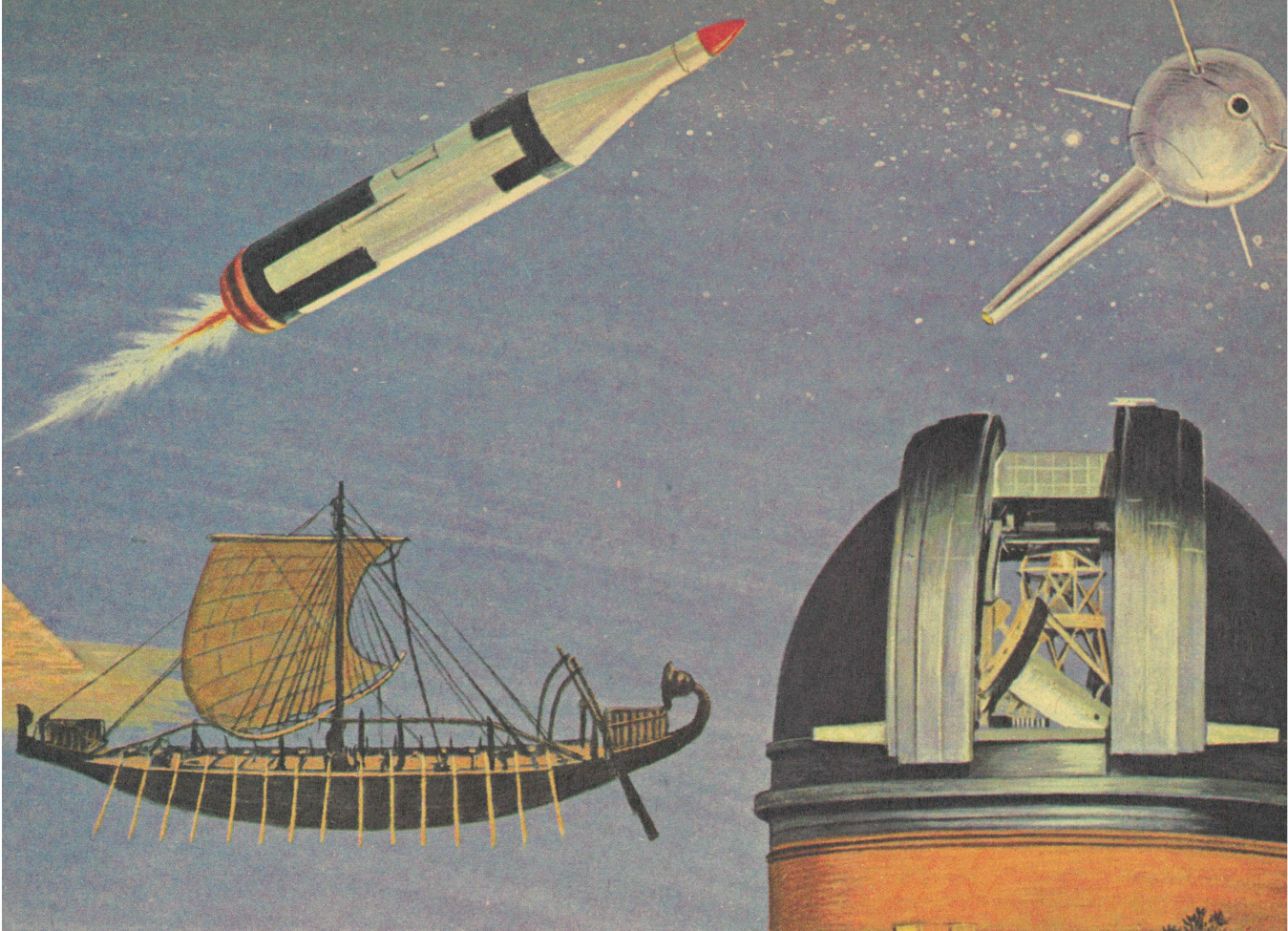
You already know many of them. Just read them — you do not need to memorize them. Refer back to these pages whenever you come to a symbol or word that is unfamiliar. You will soon find that you don't have to look back very often. Although the language of mathematics may be unfamiliar, it is simple to understand.

What is a googol?

If you were walking along the street and found a piece of paper on which was written:

$$\text{g o o g o l} < \infty$$

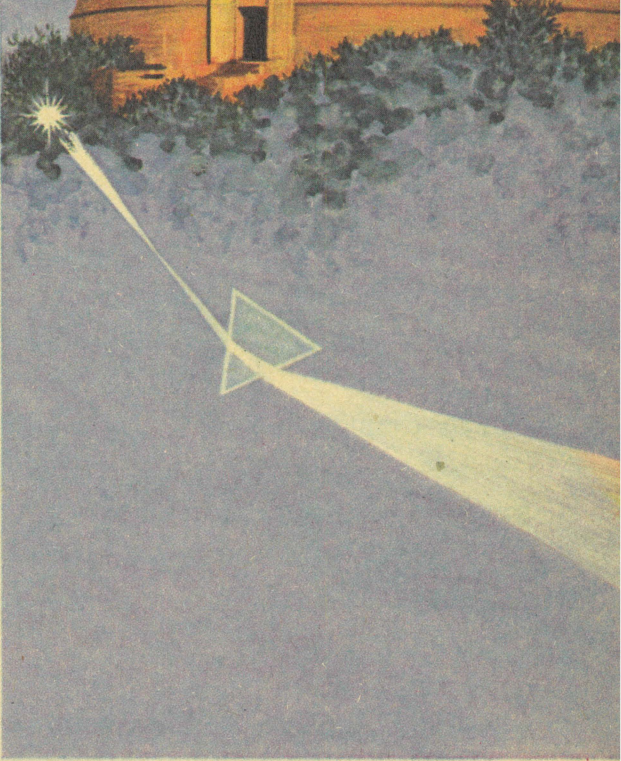
would you know that it is mathematical shorthand? The idea of using shorthand or symbols in place of words is very old and is widely used. More than 5,000 years ago, the ancient Egyptians



used symbols to stand for words. Stenographers do it today, although, of course, the symbols are entirely different. Mathematical shorthand is a short and exact way of writing mathematical instructions and quantities.

What does “googol $< \infty$ ” mean? Translated into everyday English, it reads: *a googol is smaller than infinity*. The little v turned on its side means *is smaller than*. Whatever is on the left is smaller than whatever is on the right. The figure 8, on its side, is the symbol for infinity; that is, a number greater than any we can write, speak or think.

A googol can be written as 10000-000000000000000000000000000000-00000000000000000000000000000000-00000000000000000000000000000000-000000. It is 1 followed by 100 zeros.



Mathematics is an essential part of our cultural heritage and it has played a vital role in man's history. Men had to learn mathematics to trade, count, sail, fly, build and to send a space satellite into orbit.

It is a number so large that it exceeds the number of raindrops that would fall on London, Manchester and Edinburgh in more than a century. Yet, it is smaller than infinity.

Signs and symbols are only one portion of the language of mathematics. Definition of basic terms is another. Together they make up a universal language. In this way, a scientist or mathematician writing in France or Russia can communicate precisely with his counterpart in the United States or Great Britain. Googol $< \infty$ has the same meaning to a Canadian as to a Norwegian.

What is a prime number?

Numbers are the basic tools of mathematics. If we multiply two or more numbers, we get a new number. For example, $2 \times 7 \times 11 \times 13 = 2,002$. The numbers 2, 7, 11 and 13, which we multiplied together, are called *factors* of 2,002. However, not every number has factors which are whole numbers. Let us take 13. The only numbers, or factors, we can multiply to get 13 are 1 and 13. If a number cannot be divided, or factored, by any numbers other than itself and 1, that number is called a *prime number*. How about 11 and 37? Both of these are prime numbers. Is 9 a prime number? No, because it can be factored into 3×3 as well as 9×1 .

How does algebra differ from arithmetic?

It has been said that the language of science is mathematics and the grammar of mathematics is algebra. *Algebra*

comes from the Arabic *al-jabr*, which means the reuniting of broken parts or simplification. Without it, much of our progress would not be possible. Algebra is like a tunnel cut through a mountain — a short cut or the most practical route.

One of the ways in which it differs from arithmetic is that in algebra we use numbers and letters. In algebra, we can say that a case of 50 apples = a . If a man has 10 cases, we write $10a$. Note that we omit the multiplication sign between letters and numbers. At all times the letters represent numbers. If we know the quantity, we generally use a , b , c , etc. If the quantity is not known, we usually use x , y or z . Algebra helps us find the value of unknown quantities by using known quantities.

Another basic difference between algebra and arithmetic is the number of fundamental processes we use. In arithmetic, we use addition, multiplication, subtraction and division. In algebra, we use these four but also *exponents* and *roots*.

Furthermore, in arithmetic, we usually work only with numbers greater than zero, whereas in algebra, we use numbers (or letters that stand for numbers) that are greater or smaller than zero. The numbers that are greater than zero are called *positive numbers*. Those that are smaller than zero are called *negative numbers*; they are written with a minus sign in front of them as: $-x$ or $-43b$.



What is plane geometry?

Plane geometry is a study of the world around us. It is a Greek word which means the *measure of land*. Without it our buildings would be uneven, our walls would not be perfectly upright, we would be unable to navigate through the air or on the seas. Once you are familiar with the basic language of geometry, you will see how easily everything falls into place.

Let us start with a *point*. It indicates a position in space. It does not have width, thickness or length.

A point in motion produces a *line*, of which there are several kinds:

A *horizontal line* is level with the surface of water at rest.

A *vertical line* runs up and down and is at right angles or perpendicular to a horizontal line.

An *oblique line* is neither horizontal nor vertical.

A *curved line* is always changing direction.

Parallel lines are straight lines that never meet no matter how far they are extended.

Angles are formed by two straight lines starting from the same point. Each line is called a *side* of the angle. The point at which the sides join is called the *vertex* of the angle.

One-fourth of a circle forms a *right angle*. It is $\frac{1}{4}$ of 360° , or 90° .

An *acute angle* is any angle smaller than a right angle or 90° .

An *obtuse angle* is greater than 90° , but is smaller than 180° .

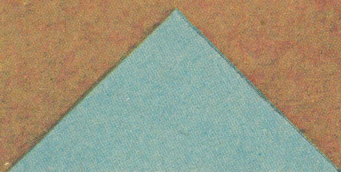
A *surface* is a line in motion. It has both length and width but no depth or thickness. A flat surface, like a desk top, is called a *plane surface*.

What is a polygon?

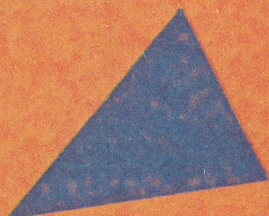
One group of plane surfaces is called *polygons*. All their sides are straight lines. If all the sides are equal, the figure is then called a *regular polygon*.



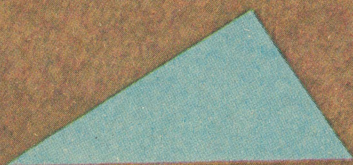
RIGHT ANGLE TRIANGLE



ISOSCELES TRIANGLE



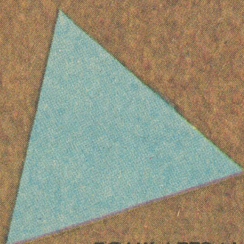
ACUTE TRIANGLE



SCALENE TRIANGLE



OBTUSE TRIANGLE



EQUILATERAL TRIANGLE

The lowest number of sides which can make a polygon is three. This is a *triangle*. There are two basic principles which mathematicians have found that apply to all triangles:

The sum of all three angles of a triangle equals 180° .

The length of the side varies according to the size of the angle opposite it. The longest side is opposite the largest angle.

Basically, there are six kinds of triangles:

A *right angle triangle* has one right or 90° angle. The longest side is opposite this angle.

An *isosceles triangle* has two equal sides. The angles opposite these sides are also equal.

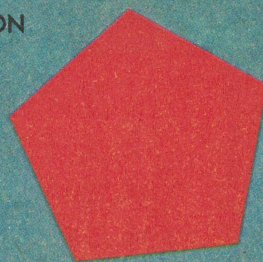
An *acute triangle* has all acute angles; that is, each angle is less than 90° .

A *scalene triangle* has three sides of different lengths.

An *obtuse triangle* has one angle that is greater than 90° .

An *equilateral triangle* has three equal sides and three equal angles.

PENTAGON



OCTAGON



HEXAGON

What are quadrilaterals?

Another type of polygon has four sides and is called a *quadrilateral*. The sum of its four angles equals 360° . The sum of the length of its sides is the *perimeter*.

There are six kinds of quadrilaterals:

A *parallelogram* has four sides. Each pair of opposite sides are parallel.

A *square* has four equal sides, and each of its four angles is 90° .

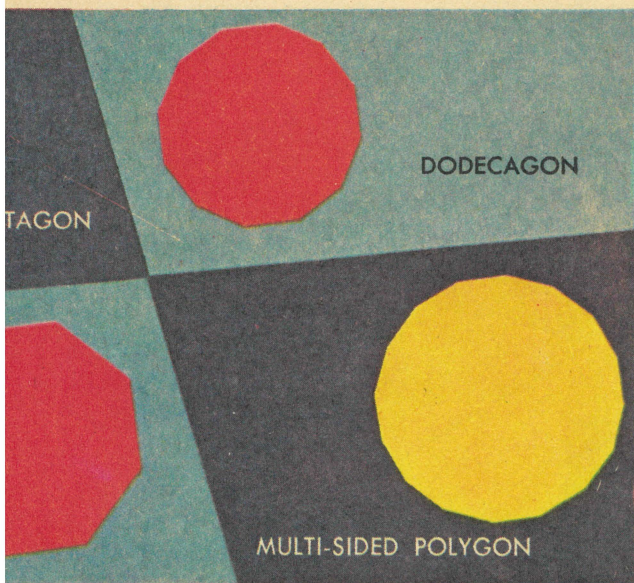
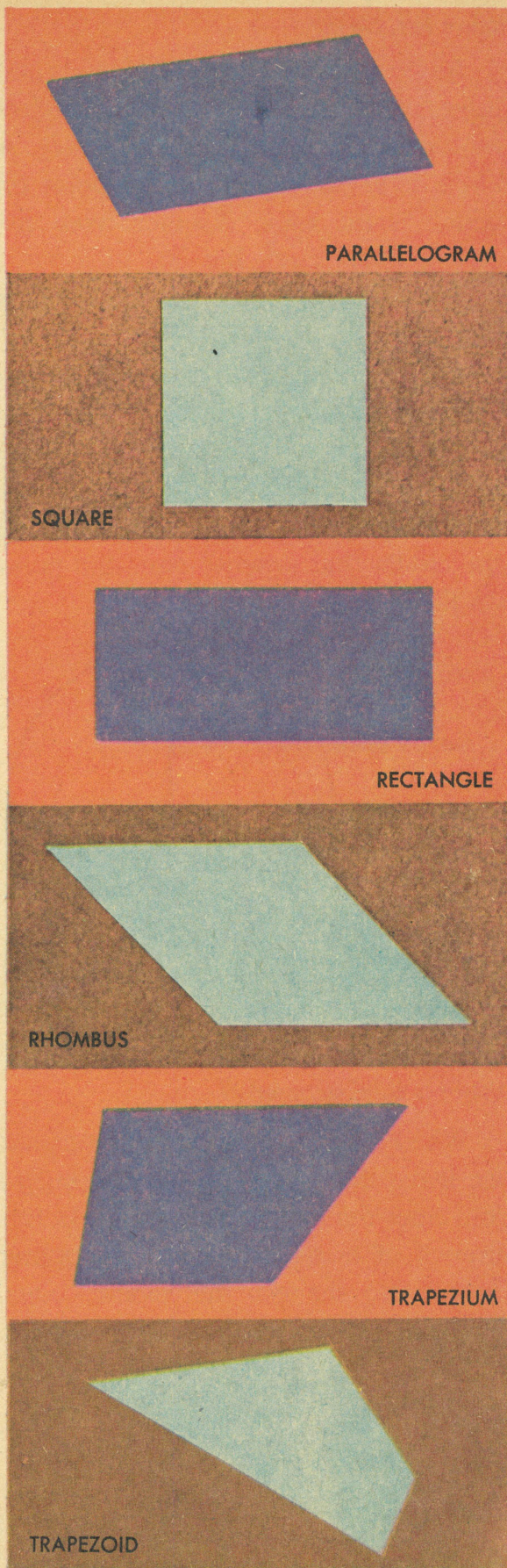
A *rectangle* is a parallelogram with each of its four angles equal to 90° .

A *rhombus* has four equal sides. Two of its angles are obtuse, or greater than 90° .

A *trapezium* has only two sides that are parallel.

A *trapezoid* has no sides which are parallel.

Polygons can have more than three or four sides. For example, a *pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; and a *dodecagon*, twelve.



What is a circle?

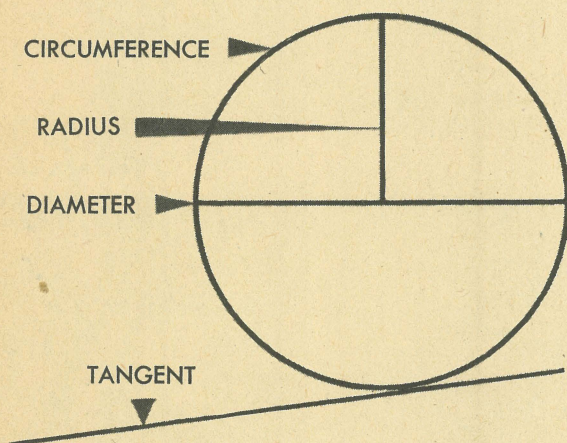
As the number of sides increases and approaches infinity, the polygon takes on a new shape. It becomes a *circle*. A *circle* is a curved line, every point on which is the same distance from the centre. Every circle has 360° .

The *circumference* is the outside boundary.

A *radius* is a straight line from the centre to the circumference.

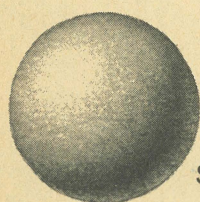
The *diameter* is a straight line through the centre of the circle.

A *tangent* is a straight line, outside the circle, that touches only one point on the circumference.



What is solid geometry?

When we add thickness to a surface, we leave the realm of plane geometry and enter that of solid geometry. In this branch of mathematics, we encounter four basic shapes: the *sphere*, *cone*, *cylinder* and *polyhedron*.



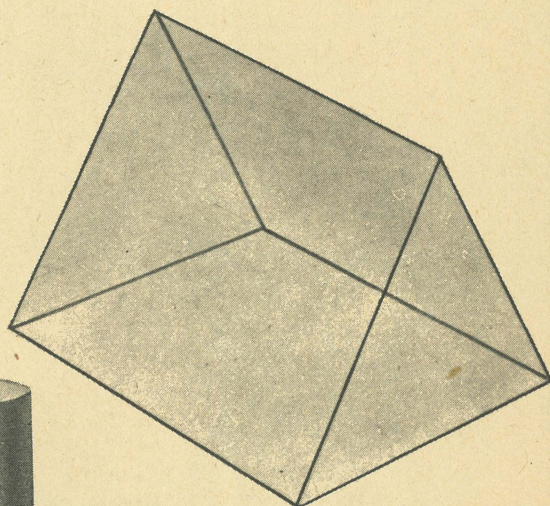
SPHERE



CONE

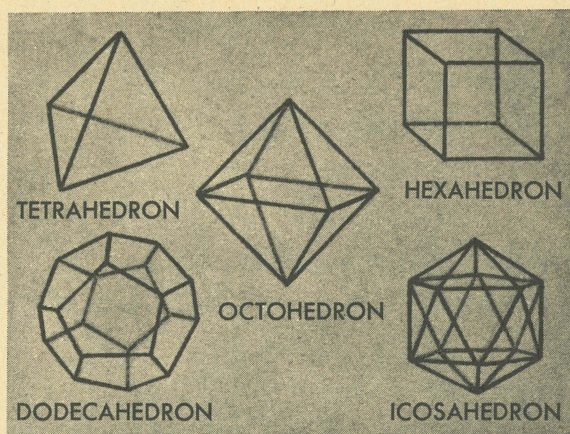


CYLINDER



POLYHEDRON

Polyhedrons are solids with length, width and thickness. Each surface or face is a polygon. There are only five regular polyhedrons:



A *tetrahedron*, or pyramid, has four surfaces or faces, each of which is an equilateral triangle.

A *hexahedron*, or cube, has six faces, each of which is a square.

An *octohedron* has eight faces. All the faces are equilateral triangles.

A *dodecahedron* has twelve faces, each of which is a pentagon.

An *icosahedron* has twenty faces. All are equilateral triangles.

Mathematical Signs, Symbols and Definitions

+	<i>Plus</i> : sign of addition $3 + 4$	π	<i>Pi</i> : used to calculate the circumference and area of a circle; it equals 3.14159
—	<i>Minus</i> : sign of subtraction $4 - 2$	'	<i>Foot</i> : used when measuring distance
\times or \bullet	<i>Multiplication</i> sign: 4×2 or $4 \bullet 2$	"	<i>Inch</i> : used when measuring distance
\div	<i>Division</i> sign: $8 \div 2$	'	<i>Minute</i> : used to measure parts of a degree; there are 60' in 1°
=	<i>Equals</i> : $2 + 3 = 9 - 4$	"	<i>Second</i> : used in measuring parts of a minute; there are 60" in 1'
\neq	<i>Not equal to</i> : $3 + 4 \neq 4 - 2$	\perp	<i>Perpendicular to</i> : forms a right angle
>	<i>Greater than</i> : $8 > 4$ or 8 is greater than 4		<i>Parallel to</i> : continues in a straight line and never meets
<	<i>Less than</i> : $4 < 8$ or 4 is less than 8		
∞	<i>Infinity</i> : greater than any number we can write, speak or think		
$^\circ$	<i>Degree</i> : unit of measurement for an angle; a complete circle has 360°		

What is the language of multiplication?

32	<i>Multiplicand</i> : number to be multiplied.
$\times 14$	<i>Multiplier</i> : number which does the multiplying.
128		
32		
448	<i>Product</i> : result of multiplication.

What is the language of division?

	19	<i>Quotient</i> : result of the division.
<i>Divisor</i> : number by which dividend is to be divided	8	$\overline{)153}$	<i>Dividend</i> : number to be divided by the divisor.
		8	
		73	
		72	
	1	<i>Remainder</i> : number left over at the end of the division if the divisor does not divide the dividend evenly.

What is the language of fractions?

$\frac{4}{9}$ *Numerator*: indicates number of parts used.

9 *Denominator*: term below the line that divides the numerator; number of parts into which the whole is divided.

$\frac{6}{7}$ *Proper fraction*: numerator is smaller than denominator.

$\frac{13}{4}$ *Improper fraction*: numerator is larger than denominator.

$2\frac{1}{4}$ *Mixed number*: a whole number and a fraction.

What are square roots, radicals and exponents?

$\sqrt{\quad}$ *Radical* indicates the root of a number. It is usually used to indicate *square root*. For example, $\sqrt{4} = 2$, since $2 \times 2 = 4$. A square root of any number is another number, which, when multiplied by itself, equals that number. Here is another example: $\sqrt{16} = ?$ What number can we multiply by itself to give us 16? The answer is 4, since $4 \times 4 = 16$.

$\sqrt[3]{\quad}$ When the radical is used with a small number in the upper left, it stands for a smaller root. For

example $\sqrt[3]{27} = 3$, since $3 \times 3 \times 3 = 27$. In the same way, $\sqrt[4]{256}$ stands for the fourth root of 256, or a number multiplied by itself four times to equal 256. The answer is 4, since $4 \times 4 \times 4 \times 4 = 256$.

A small number written at the upper right-hand corner of another number is called an *exponent*. It is mathematical shorthand to indicate the number of times you have to multiply the number by itself. For example: $2^2 = 2 \times 2$ or 4. If we wrote 3^4 we would mean $3 \times 3 \times 3 \times 3$, or 3 multiplied by itself four times.

Numbers and Numerals

We do not know when primitive man first began to use speech instead of sign language to communicate with his family and neighbours, but we do know that man used words for thousands of years before he learned how to set these words down in writing. In the same

way, many thousands of years passed after man learned to name numbers before he began to use signs for these numbers — for example, to use the numeral 3 in place of the word *three*.

Men needed numbers and had to learn to count. Maybe it started when

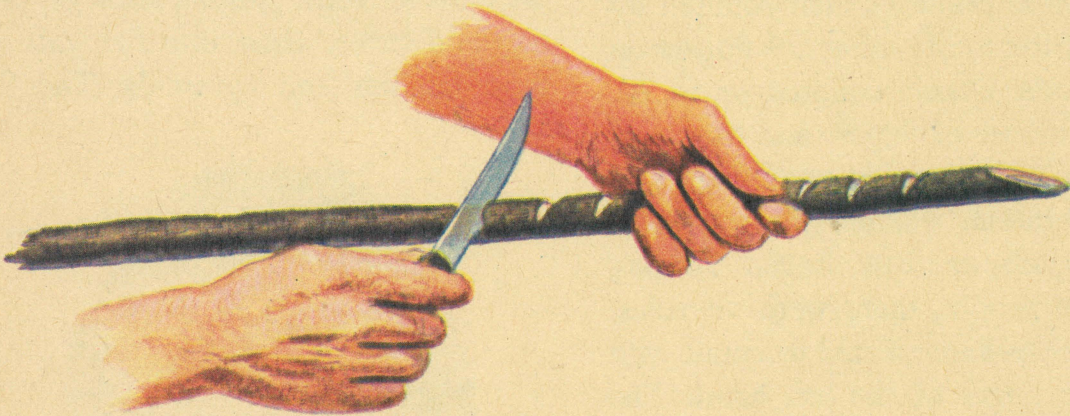


Early man knew "one" and "two." Some primitives (left) used "many" to signify three.

Many primitives used pebbles as counters. Peruvian Incas (below) used knotted ropes.



Notching a stick as a counting tool came from early history. Each notch indicated a single mathematical unit.



one cave man wanted to trade the sabre-tooth tiger he had slain for his neighbour's three spears. Or maybe it started when a twelve-year-old cave-man junior wanted to tell his brothers and sisters about the four hairy mammoths he had seen during the hunt.

What is a number?

At first, primitive man used sign language to indicate the number he wanted to use. He may have pointed to the three spears in his neighbour's cave or to the slain sabre-tooth tiger at his feet. He may have used his fingers to indi-

cate the number. Two fingers raised on one hand meant *two* whether he was talking about two spears, two sabretooth tigers, two caves or two arrowheads.

In everyday usage, we understand that a *number* is a word or a symbol which signifies a specific quantity and it is not necessary to define what we are talking about. For example, *three* or 3 can refer to three airplanes, three pens or three schoolbooks.

How did primitive man count?

Some primitive men did not use numbers beyond two. Only a century ago, when explorers visited the Hottentots in Central Africa, they found that these people had only three numbers: *one*, *two* and *many*. If a Hottentot had three or more cows, even if he had 79 or 2,000, he would count that number as *many*. Most primitive men counted up to 10, or the total number of fingers on their hands. Others counted up to 20, or the number of fingers and toes.

When you count on your fingers, it does not matter whether you start with your thumb or little finger. Among primitive people, there were set rules. The Zuni Indians started to count with the little finger of the left hand. The Otomacs of South America began with the thumb.

As men became more civilized, they used sticks, pebbles and shells to write numbers. They set three sticks or pebbles in a row to show that they meant three. Others made notches in a stick or tied knots in a rope as a means of writing their numbers.

When did man first use numerals?

The earliest written numbers so far discovered were used in ancient Egypt and Mesopotamia about 3000 B.C. These people, living many miles apart, each independently developed a set of numerals. Their simple numerals, 1, 2, 3, were copies of the cave man's sticks or notches. It is interesting to note that in many of the numeral systems found throughout the world, 1 was written as a single stroke (like a stick) or as a dot (like a pebble).

EGYPTIAN



BABYLONIAN



EARLY ROMAN



CHINESE



EARLY HINDU

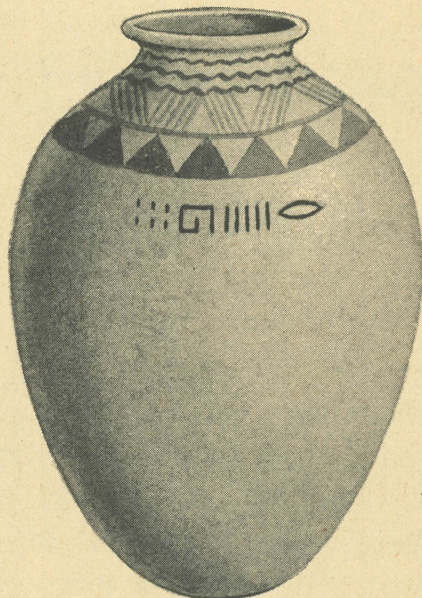


MAYA



How did ancient man write?

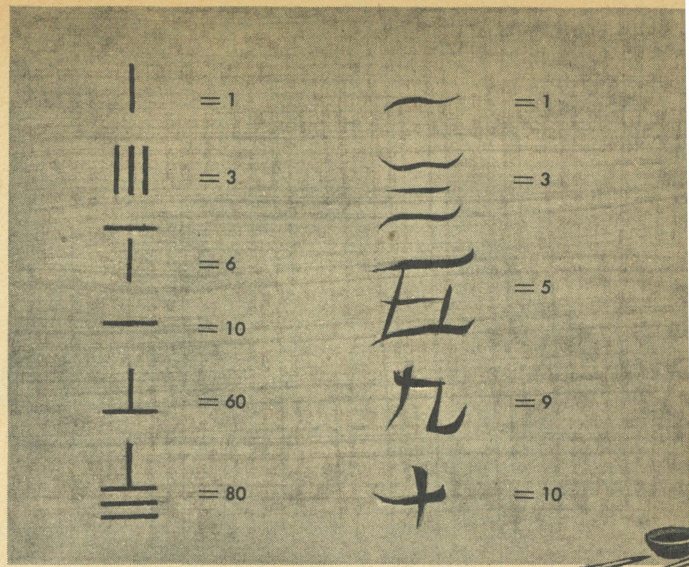
The ancient Egyptians wrote their numerals on papyrus, a special paper made from reeds, painted them on pottery and carved them into the walls of their temples and pyramids. The Su-



Numbers on Egyptian vases often told their contents.

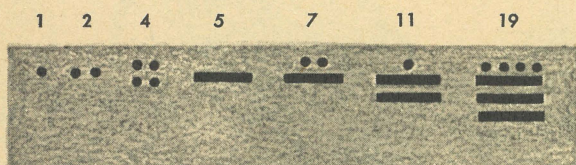


The Babylonians wrote their numbers on clay tablets. The Chinese wrote numbers with brush and ink on silk.



merians taught the Babylonians how to cut their numerals into soft clay tablets. The ancient Chinese did their number writing with ink and a bamboo brush or pen on cloth.

In the Western Hemisphere, with no contact with the rest of the world, the Mayas of Central America developed one of the most remarkable of the early number systems. They made their numerals using only three symbols: a dot •, a straight line —, an oval (V) .



How long would it take you to write a million?

The ancient Egyptians, Babylonians and Chinese, like the early Greeks and Romans, used special signs or numerals to express large numbers. This development of special signs for large numbers was the first advance in numeral writing. Imagine the difficulty and the time needed to write one million by cutting

notches in branches or setting pebbles out in the sand. If you were to follow these methods, or count pennies one at a time (1 a second), it would take you 278 hours, or 11 days and 14 hours of nonstop counting to reach one million.

How did ancient man write large numbers?

Here are several examples of how the ancient people wrote their large numbers:

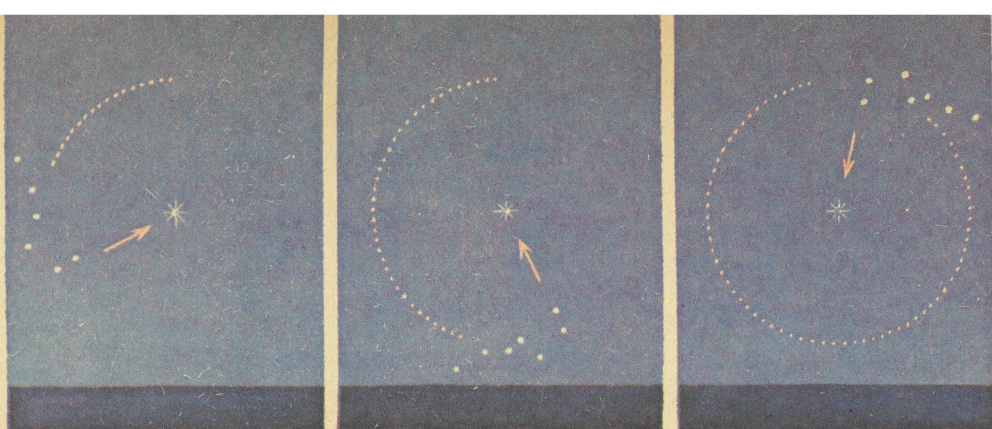
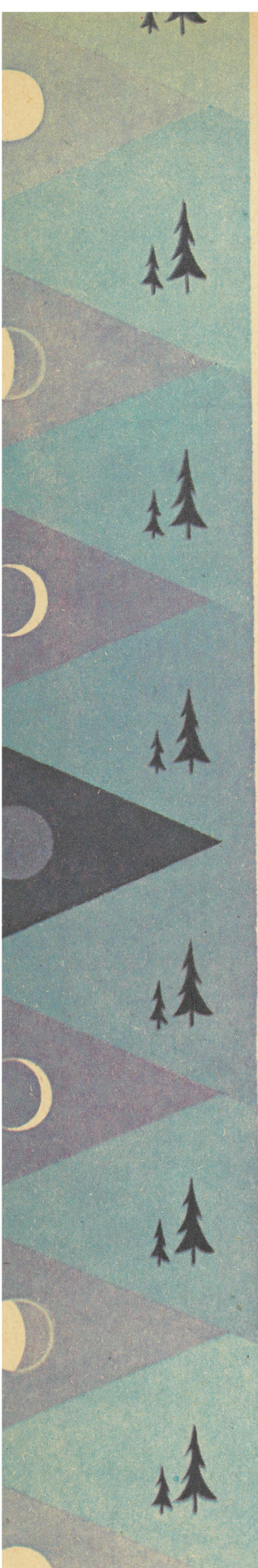
The ancient Egyptians wrote 100 as ⑨ and 1,000 as ①.

The ancient Babylonians used a more complex system. Their numeral for 50 was 𐎶𐎵𐎺. This was made of 60 𐎶, their symbol for minus 𐎵, and their numeral for 10 𐎺.

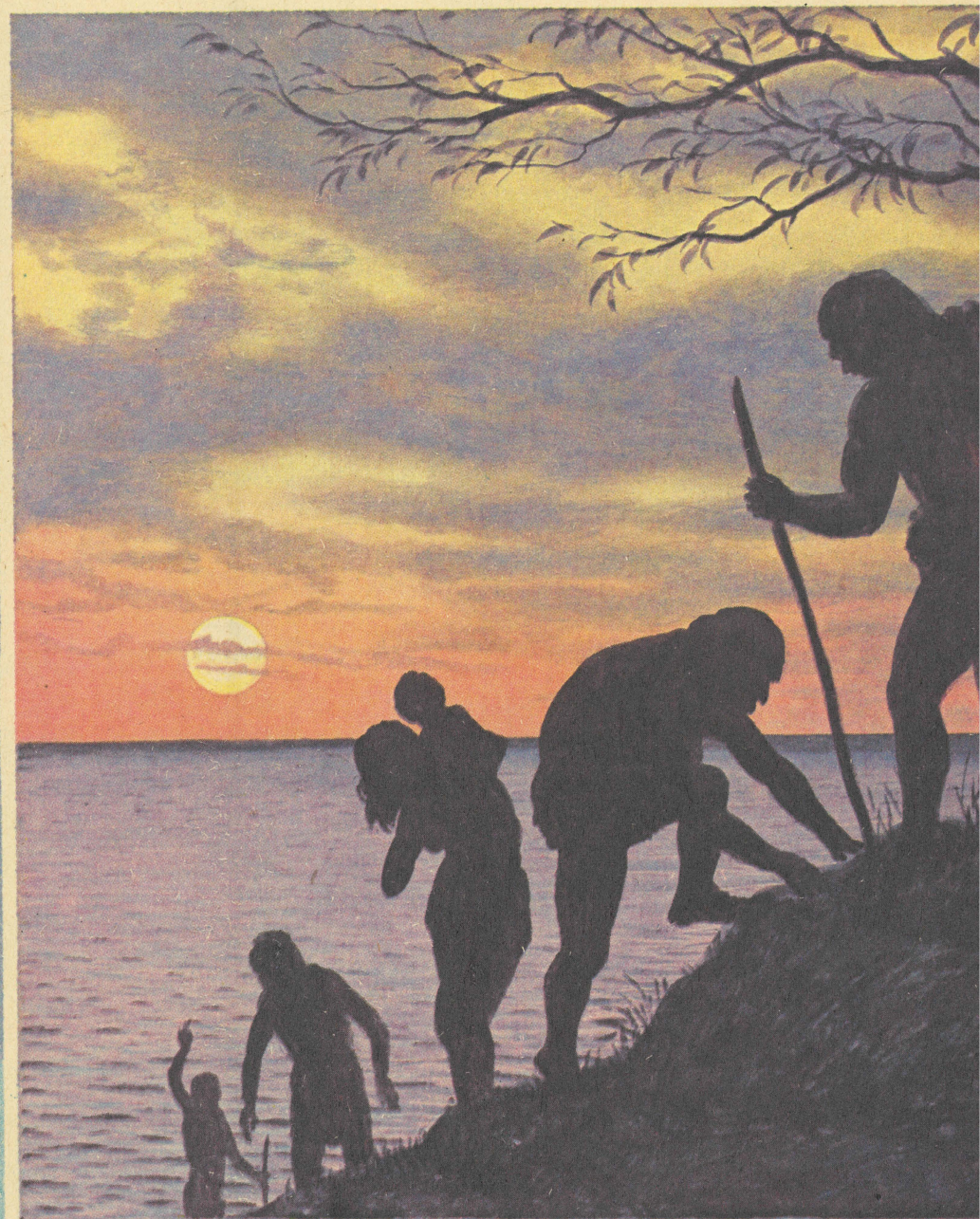
In the ancient Chinese numeral system, the symbol for 100 was 百 while 千 indicated 1,000.

The early Romans wrote 100 as C and this was also used by the later Romans. The early Roman numerals for 1,000 was M̄ but this was changed to M by the later Romans.



Placing the symbols vertically, one



The Big Dipper revolves around the North Star. One complete revolution takes a day.



Early man found his way home at night by walking in the direction of the setting sun. The rising and setting of the sun was one of his earliest ways of telling direction.

under the other, was the method used by the Mayas to write large numbers. This positioning meant multiply. Their symbol for 100 was  or 5 — times 20 .

Why are numerals important?

Every forward step in civilization brought additional uses for numerals. If a man owned land, he wanted to

measure his property. If he sailed in his boat, he wanted to know how far from shore he was. If he wanted to build a temple or pyramid, he had to know how many stones he would need. When he learned to calculate with his numerals, he could measure time, distance, area and volume. By using numerals, he increased his knowledge and control of the world around him.

Mathematics in Early History

How did man first tell direction?

Early man had no towns or villages and he wandered about to hunt for food. Since there were neither roads nor maps, he had to rely on the sun and stars to guide him.

Some men, living along the coast, saw the sun come up from behind the mountains and disappear later into the water. They learned that by walking towards the rising sun, they could reach the mountains, and that by walking towards the setting sun, they could return to shore.

By watching the heavens at night, they found that groups of stars remained together as they moved across the sky. In the Northern Hemisphere, the stars moved in a circle around a fixed point, the North Star. Early man used that star as a guidepost.

How did early man measure time?

The moon was man's first calendar. The moon waxes and wanes; from being almost invisible, it grows to a full, round globe, and then gradually disappears again. Men found that it took about 12 moons, or 360 days, for the seasons to complete one cycle. This was the first measure of the length of the year.

The connection between the regularity of the seasons and the position of the sun and stars was the next observation used to measure the passage of time. By locating the position of a certain star on the eastern horizon just as the sun was setting, it was possible to measure the length of the year more accurately. As early as 4000 B.C., the ancient Egyptians had already set the year at 365 days.



Shadow clocks were the earliest "timepieces." The obelisk or post cast the shadow of the sun. Then the space between the shadow of the rising sun and that of the setting sun was divided. These divisions became the hours.

What is a shadow clock?

The shadows cast by the sun during the day became the first clock. Shadows grow shorter and change direction as the sun rises. Later in the day, the shadows grow longer again. Therefore, the time of day could be fixed by the direction and length of the sun's shadow.

The first shadow clocks, or sundials, were crude. They consisted of an upright stick stuck into the earth, or one stone, or an obelisk mounted on a stone base. These early clocks were both unreliable and inaccurate. If the sun didn't shine, the clock could not be used. Furthermore, since the length of the shadow varied with the season, it was difficult to tell time exactly. It was not until thousands of years later that the sundial was perfected by Moorish mathematicians.

Who were the first practical mathematicians?

The ancient Egyptians, who lived more than 5,000 years ago, are some-

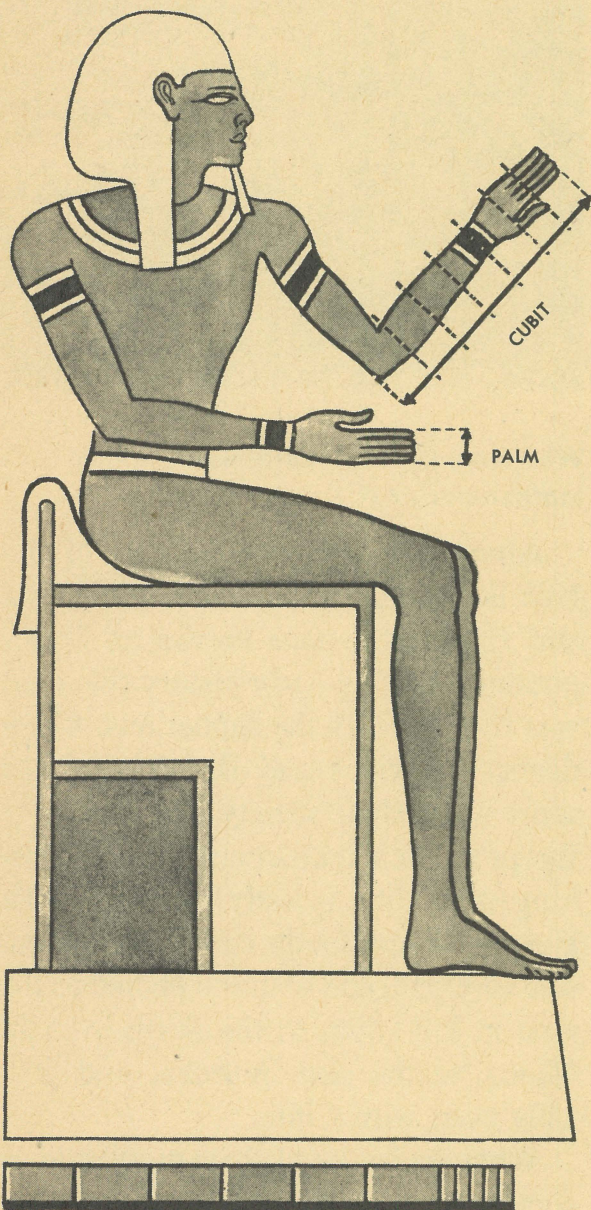
times called the practical mathematicians of antiquity, but by modern standards, their mathematics was extremely elementary. At the time they were starting to design and build the pyramids, the Egyptian mathematicians still counted on their fingers. All their arithmetic was a form of either addition or subtraction.

Nevertheless, they did make remarkable contributions to our knowledge of mathematics. Their priests, who were mathematicians, directed the building of temples and pyramids, which served as tombs for the pharaohs. These priests were the architects and engineers who made plans similar to the blueprints used today. These plans called for exact measurements. The crude measurements used by primitive men were not precise enough for the temple and pyramid builders.

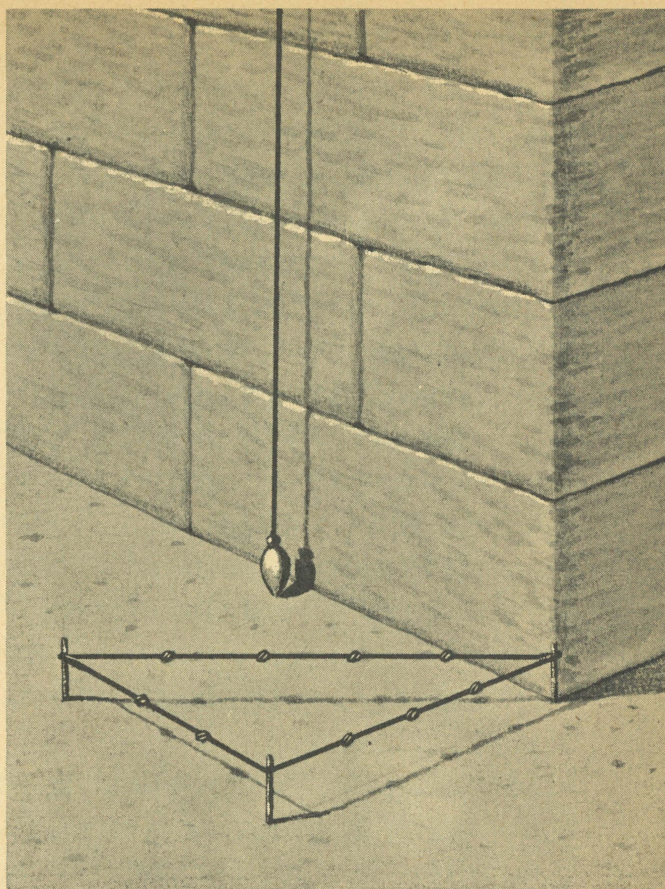
How long is a cubit?

To obtain the necessary precision, the Egyptians established a system of

measurement based on the human body. The main unit was a *cubit*, the distance from the elbow to the finger tips. Each cubit was divided into seven *palms*, and each palm was divided into four *digits*. According to modern measurements, a cubit equals 18 to 22 inches.



The Egyptian priests made metal bars to correspond to a cubit, with the smaller divisions, palms and digits, indicated on the bars. Today, we would call the bar a *cubit ruler*, marked off in palms and digits, just as our yardstick is marked off in feet and inches.



Egyptians used the plumb line and knotted triangle.

How do you make a square corner?

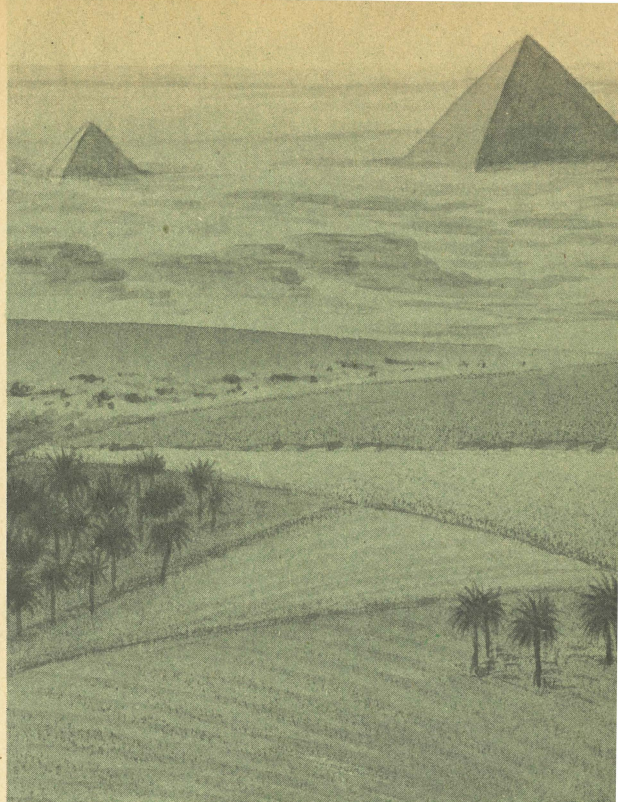
One of the most difficult problems in building the pyramids and temples was to make the base perfectly square. An error would mean that the entire building would be out of shape. The Egyptians solved this by making a *building square*. The first one was probably made by holding a piece of string, with a weight attached to the bottom, over level ground. The string would be perfectly plumb or vertical, and the angle between the plumb line and ground was a perfect right angle. These builders also discovered how to use measuring ropes with equally-spaced knots to make right-angle triangles as guides to building square corners.

How did they find the area?

Another problem which the temple and pyramid builders faced was the calculation of area, or how much flat surface is contained within a boundary. Why or when the square was used to measure area is uncertain. Maybe the first clues came from laying square tiles on the floors of the temples. If one room was 8 tiles wide and 8 tiles long, they saw that the room required 64 tiles to cover the floor. Another room 8 tiles wide and 10 tiles long required 80 tiles. From this they learned that the area of a square or rectangle was equal to the width multiplied by the length, or $\text{Area} = \text{width} \times \text{length}$.

Measuring the land required more knowledge of mathematics. The priests surveyed or measured the land because the amount of taxes depended upon the size of the farm, and because the annual flooding of the Nile washed away all boundary stones, making it necessary to measure each farm again and again, year after year.

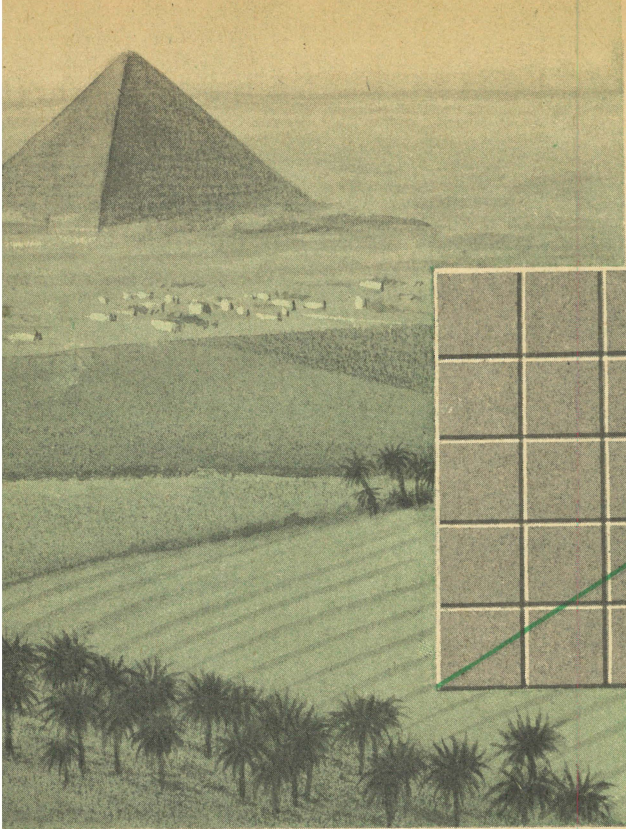
These farms could not be divided into squares or rectangles easily. They could, however, be divided into triangles. Either by accident or after prolonged study, the Egyptians found that a square or rectangle could be divided into two equal triangles. With this clue, they learned to measure the area of any right-angle triangle. Its area was one-half of the base multiplied by the altitude or height, or $\text{Area} = \frac{1}{2} \text{ base} \times \text{altitude}$. Many years passed before they discovered that the same formula could be used for any triangle, even if it had no right angle.



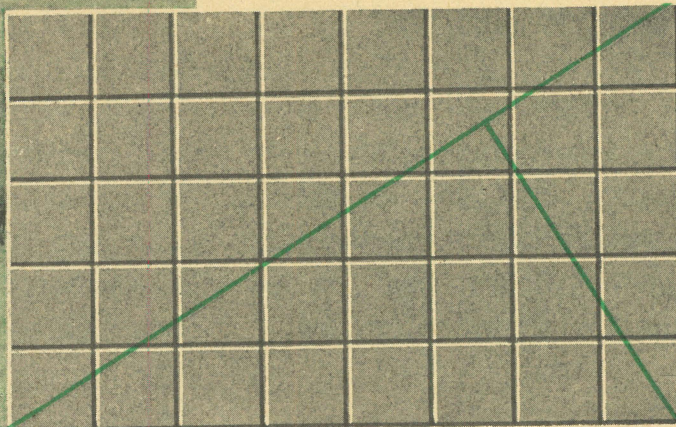
Who were the Mesopotamian mathematicians?

About a thousand miles east of the Nile lies the fertile valley of the Tigris and Euphrates, once known as Mesopotamia. During early history this land was the home of the Sumerians, Chaldeans, Assyrians and Babylonians. In some ways, their society was similar to Egypt's. Their mathematicians also belonged to the priestly caste. Unlike Egypt, Mesopotamia carried on extensive foreign trade with the people to the west in Lebanon, to the north in Asia Minor, to the east in India, and possibly even with China.

What we know of their mathematics comes to us from the baked clay tablets on which they wrote. The Babylonians had advanced mathematical knowledge as far back as 2500 B.C. We know that they inherited from the Sumerians their cuneiform, or wedge-



Egyptian priest-mathematicians (left) surveyed the land after the annual Nile floods, dividing it into triangles.



Man's awareness of the relationship between triangles and rectangles was a step in his quest for knowledge.

shape, writing and numerals. We are indebted to these people for several of our basic mathematical concepts and notations.

When were decimals first used?

The Babylonians introduced the *position system* of numeral writing on which our decimal system is based. We know that the value of any digit depends upon its position in the number. For example, we read 2 as *two*, and 20 as *twenty*. Placing a 2 in the second position to the left means that we automatically multiply it by 10. The ancient Babylonians wrote their numbers in somewhat the same way. Thus, \gg stood for two and $\gg\text{z}$ for 20. Although they introduced the position, or decimal system, into mathematics, it was not until the ninth century A.D. that this system was introduced into Europe by the Saracens.

Who introduced the zero?

It was the Babylonians who first used a symbol for zero. While we have no records of the symbol being used during the early period, there are clay tab-



Babylonian tables included multiplying up to 60×60 .

lets dating back to 200 B.C. on which the symbol is used to designate the absence of a figure. In their multiplication tables, which contained all the numbers up to 60 multiplied by 60, they used 𐎶 as a zero. Since the Babylonians traded with India, it is believed that this concept is the basis for today's zero, which had its origins in India. It was the Moors and Saracens who brought the zero into Europe about the ninth or tenth century A.D.

What is the sexigesimal system?

Another Babylonian contribution to our mathematical heritage, one which has remained in use to this day in astronomy and geometry, is the *sexigesimal* system, which is based on 60. They used this system for their weights and measures. The division of our year into 12 months, the hours into 60 minutes and the minutes into 60 seconds, is attributed to the Babylonians. So is our division of a circle into 360° .

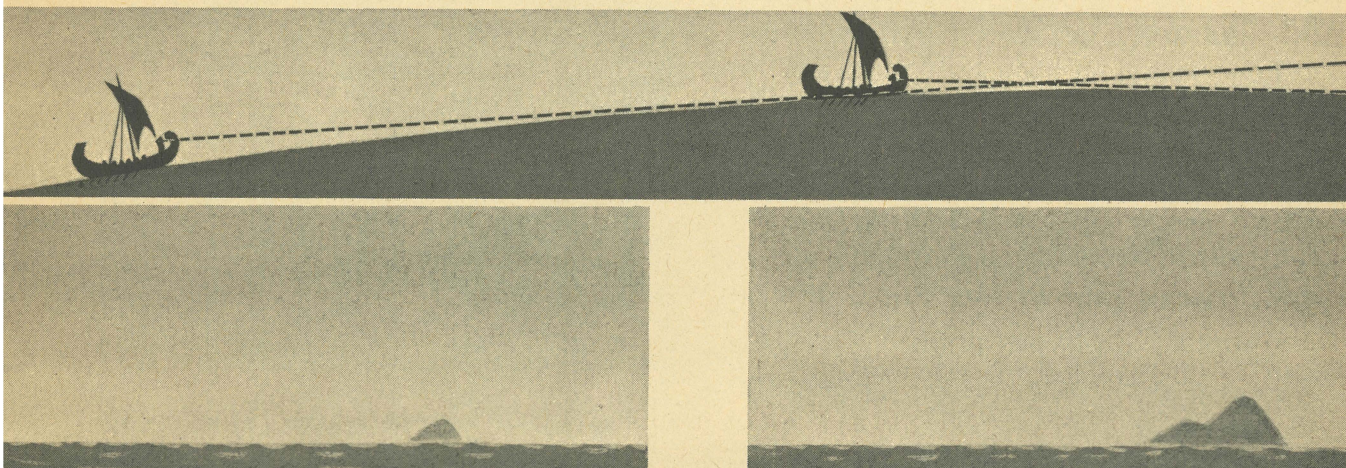
Sailors, Sun and Stars

About 500 miles northeast of Egypt and some 500 miles northwest of Mesopotamia lies the Syrian coast along the Mediterranean Sea. It was here, in the ancient land of Phoenicia, that a seafaring nation thrived more than 3,500 years ago. From the ports of Tyre and Sidon, the Phoenician seamen sailed the Mediterranean. About 3,000 years ago, their ships had passed through the

western end of the Mediterranean and undoubtedly sailed northward up toward Great Britain and southward down along the western coast of Africa.

Although their small boats were sturdy, they sailed close to shore to remain near known landmarks. In time, they ventured into the open seas, but only after they had developed the necessary mathematical navigation.

The early Phoenician sailors were among the first to realize that the world was not flat, but curved.



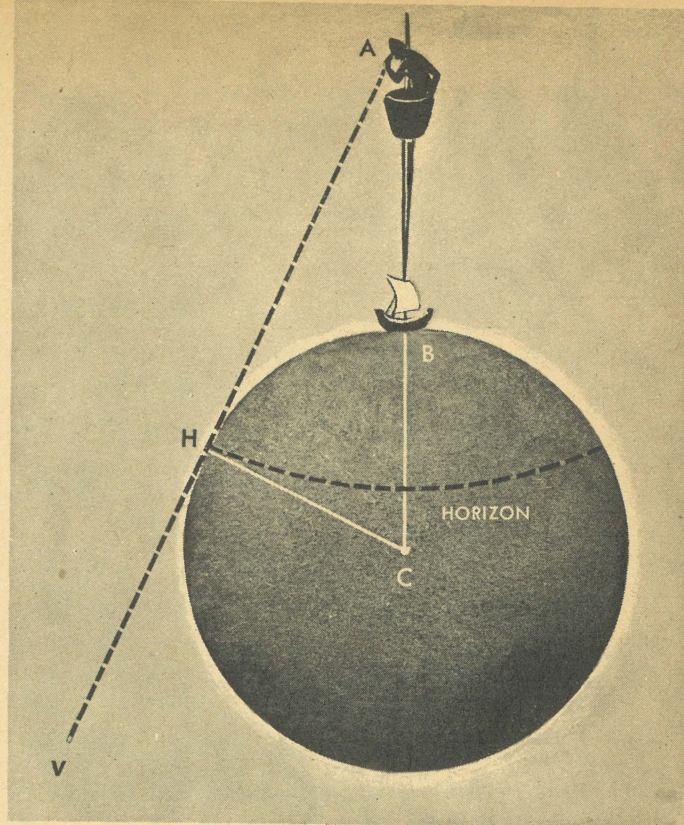
How did the Phoenicians navigate?

The Phoenician seamen, who travelled day after day, never found the ends of the earth. In their ports they could see the tops of the masts of ships as they approached the harbour. Then they could see the sails and finally the entire boat as it came closer to land. At sea, high on the mast, a sailor could see distant landmarks not visible to those on the deck below. They soon realized that the earth was not flat, as many people believed in other civilized parts of the ancient world — it was a sphere. It was many hundreds of years later that the Greeks and Romans used this knowledge to make measurements at sea.

How can you find the distance to the horizon?

This knowledge works for land measurements as well. When you stand on a mountain or a tall building, you have the same view as the Phoenician sailor high in a crow's nest on the mast. How far is *far away*?

Suppose we draw an exaggerated



view of you, high in the crow's nest looking out as far as you can see, and we'll also draw the earth.

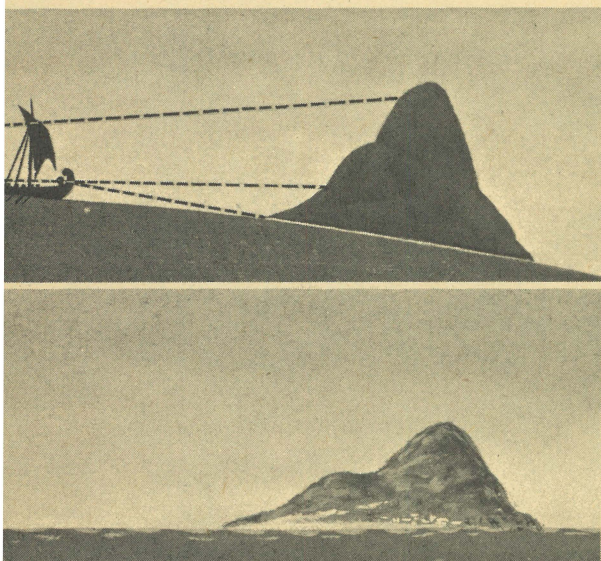
You are at point A in the diagram.

Your distance above the earth = AB (you are at A and the boat at B).

The line BC is the radius of the earth, a line from the centre to the surface. It is about 4,000 miles.

You look toward the point where the sky appears to meet the water, along the line AHV. This line touches the earth at one point only. It cannot touch in two places because this would mean you are looking through the earth. You can turn around and look in any direction, but your line of sight will touch the earth at only one place. If you turn and look in every direction, those individual places would form a circle on the sphere of the earth. We call that circle the *horizon*.

To find the distance to the horizon,

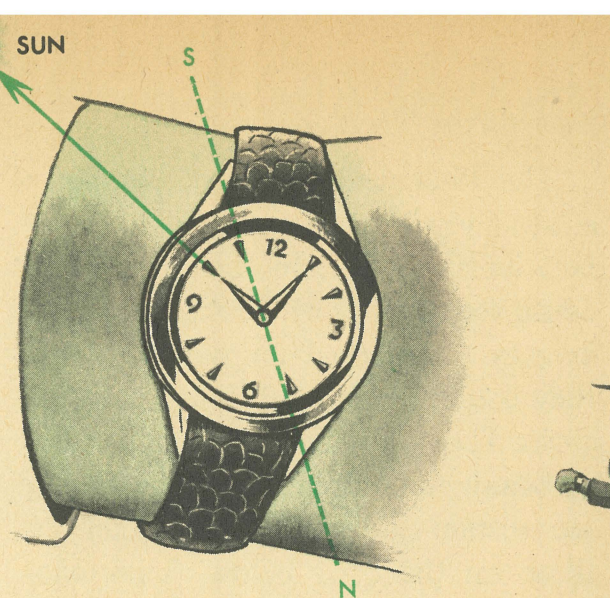


we use the mathematical formula:
 $d = 89.443 \sqrt{h}$ where d = distance to horizon and h = height above the earth's surface measured in miles. Let's try a problem.

You're in a balloon flying 4 miles above the earth. How far is the horizon? We use the formula: $89.443 \sqrt{4}$. The square root of 4 is 2 ($2 \times 2 = 4$) and our answer is $89.443 \times 2 = 178.89$ miles.

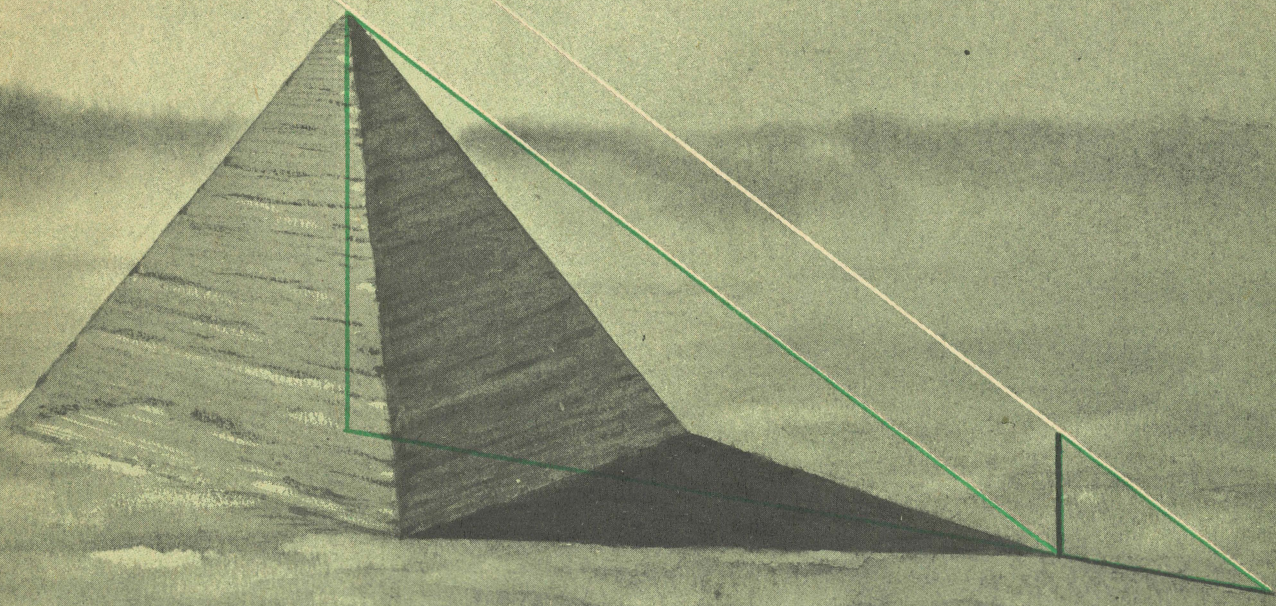
How can you use your watch as a compass?

Hold your watch so that it is level with the ground and point the hour hand toward the sun. South is halfway between the hour hand and the 12 o'clock mark. For example, at 5 min-



utes after 10 A.M. (Standard Time), with the hour hand pointing at the sun, halfway between the 10 and 12 — or at the 11 — is the location of south. An imaginary line drawn through 11 and 5 points north and south.

Thales showed it was possible to calculate the height of an object by measuring its shadow and comparing it with the shadow cast by a measuring stick.



The Contribution of the Greeks

There is a belief that mathematics did not become a science until the Golden Age of Greece. Although the Egyptians, Babylonians and Phoenicians brought mathematics a far distance from the days of primitive man, they were interested only in practical math, the computations needed for everyday living, for building, for sailing, for trading. They applied mathematics to specific problems with little or no thought given to basic theories and underlying rules. It was the Greeks who took the giant step from the practical to the theoretical.

Our knowledge of Greek mathematics begins about 600 B.C. when Thales, one of the seven wise men of Greece, introduced the study of geometry into Greece. The Egyptians knew how to measure the height of a pyramid by its shadow, but it was Thales who formulated the basic rule and proved it works in all cases. Demonstrating that a rule is true under all conditions is what mathematicians call *proof*.

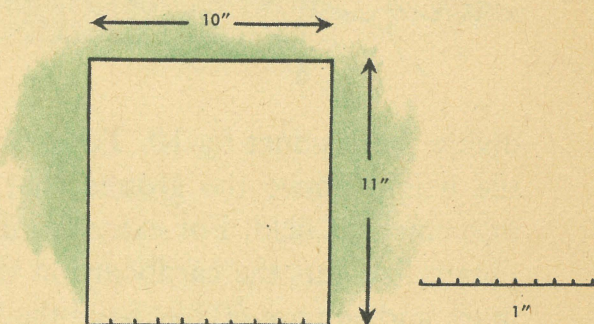
How can you measure any height?

You can measure the height of anything using the principles of Thales. All you need is a simple *measuring instrument* which you can make out of cardboard and a piece of wood.

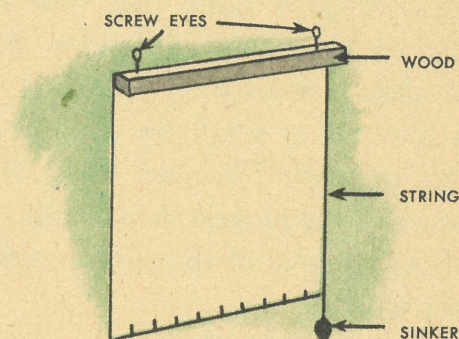
Cut a piece of cardboard 10 inches wide and 11 inches high. Starting along the bottom edge at the right, measure off in inches. If you want more accurate results, divide the space between inches into tenths, using the scale in-

cluded in the diagram. Number the inch marks.

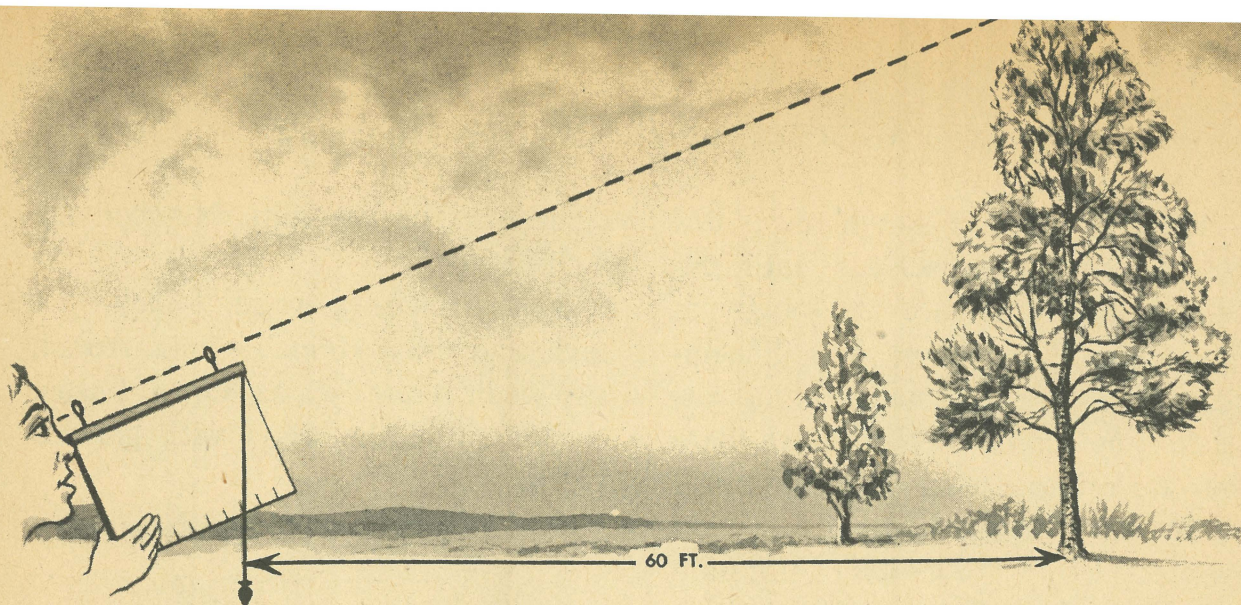
Take a piece of wood, 1 inch \times 1 inch, and about 10 inches long. Attach two small screw eyes as shown. Fasten cardboard to the wood with glue or tacks.



Take a piece of string, 15 inches long, and attach a nail or fishing line sinker to one end. Tie the other end under the wood just at the right-hand edge of the cardboard. You now have a measuring instrument.



To tell the height of an object, sight through the two screw eyes so that you can see the exact top of the object. The string will hang vertically or plumb. Note the number on the scale that the string touches. Next measure the distance to the object. Multiply this distance by the number on the scale and



divide the product by 10. To this, add the height above the ground that the instrument is held. For example, if the plumb line cuts the cardboard at the 5 mark and you are 60 feet from the tree, multiply $5 \times 60 = 300$. This is divided by 10 so that you get 30. If the instrument is 3 feet above the ground, then the height of the tree is 33 feet.

What is the Pythagorean Theorem?

Pythagoras, who lived about 500 B.C., discovered a basic rule that applies to all right-angle triangles, and proved that it works in all cases, no matter how big or small the triangle is. The *Pythagorean Theorem* states that the square of the longest side equals the sum of the squares of the shorter sides. In other words:

$$3^2 + 4^2 = 5^2$$

$$3 \times 3 + 4 \times 4 = 5 \times 5$$

$$9 + 16 = 25$$

Many of the rules of Greek geometry have come to us from the "Elements," which was written by Euclid about 300 B.C. In its translated form, this textbook was used in many of our schools until about fifty years ago.

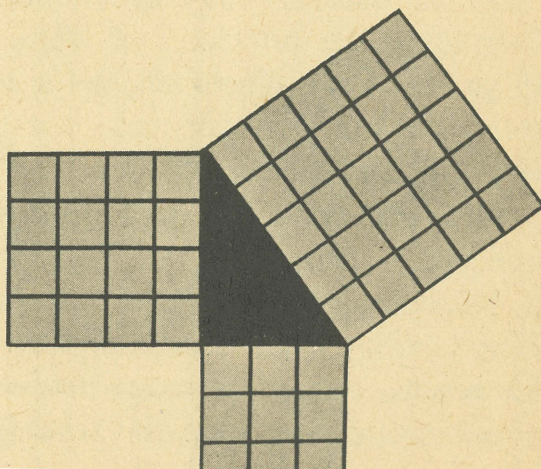
This simple measuring instrument, like the surveyor's theodolite, can help you measure the height of any object. Follow the directions in the text to see how.

How was the earth's size first measured?

The Greek mathematician, Eratosthenes, who lived about 225 B.C., was librarian of the great library at Alexandria in Egypt. He is the first man known to have measured the size of the earth. He applied mathematics to two observations:

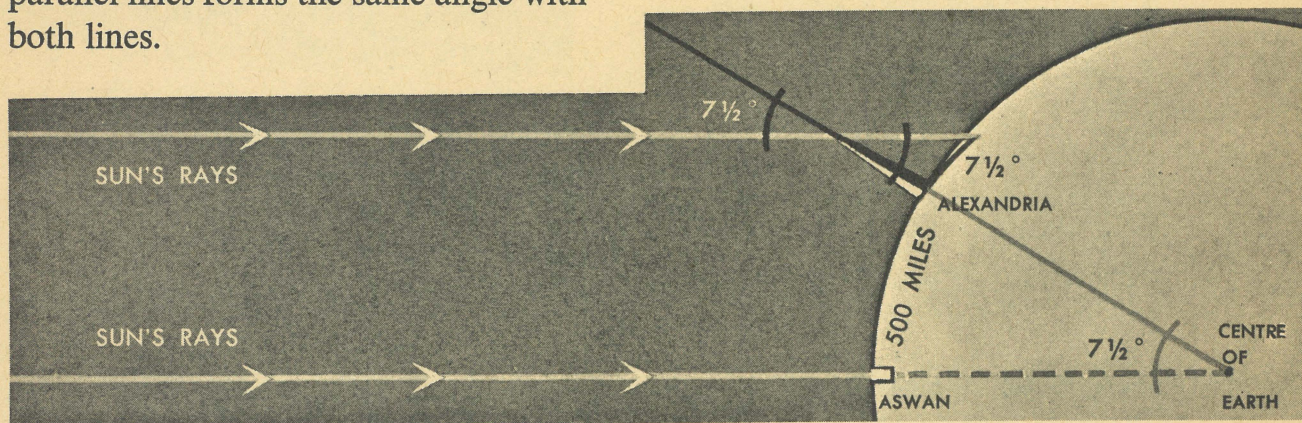
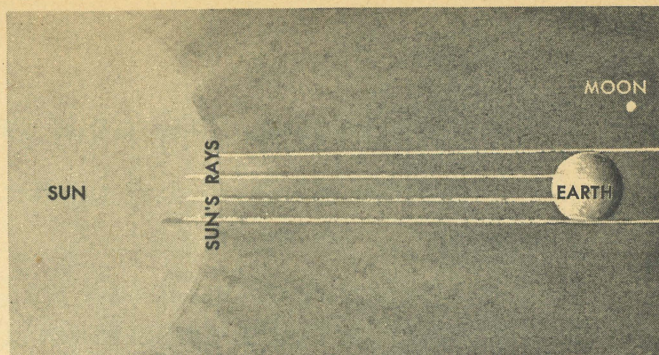
At Aswan, near the first cataract of the Nile, it was possible to see the reflection of the sun in a deep well since the sun was directly overhead and cast no shadows on a certain day of the year.

Square of longest side = sum of squares of shorter ones.



At the same time on the same day, the sun cast a shadow of $7\frac{1}{2}^\circ$ in Alexandria, some 500 miles to the north.

Eratosthenes was able to compute the circumference of the earth by using two geometry proofs which earlier Greek mathematicians had developed. First, it was known that opposite angles are equal, and, secondly, it was proven that any straight line that crosses two parallel lines forms the same angle with both lines.

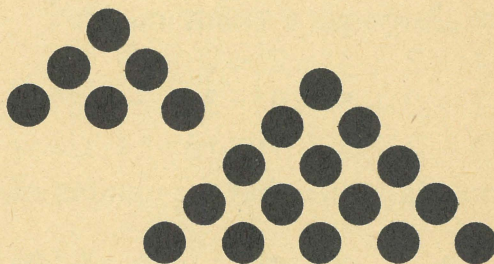


Furthermore, Eratosthenes knew that a circle has 360° . He also knew from his measurements that $7\frac{1}{2}^\circ$ was equal to 500 miles on the earth's surface. Since $7\frac{1}{2}$ goes into 360 (the number of degrees in a full circle) 48 times, he multiplied 48 by 500. He computed the circumference at 24,000 miles. With today's precision instruments, we have calculated the earth's equatorial circumference at 24,902.3786 miles.

only 11,143 miles off, since the moon is 238,857 miles from the earth.

What are triangular numbers?

Other Greeks explored the magic of numbers. When adding consecutive numbers, the students of Pythagoras found that they could make rules about their totals. Consecutive num-



When did man learn the distance to the moon?

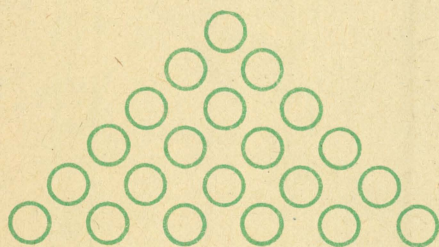
In the second century B.C., Hipparchus, the renowned astronomer of Alexandria, figured out the distance from the earth to the moon. His calculations showed the moon to be about a quarter of a million miles away. He was

bers formed triangles. To find the sum of any group of consecutive numbers, they created the formula:

$\frac{n(n+1)}{2}$ = the sum, where n is the value of the last consecutive number, where the first number is 1.

What is the sum of the first six numbers? Let $n = 6$ and using the formula, we find:

$$\frac{6(6+1)}{2} = \frac{6 \times 7}{2} = \frac{42}{2} = 21.$$



What are square numbers?

You can use checkers or marbles to form any square you wish. The smallest square has one checker or marble. The second square has two checkers in each line. The third had three in each line, etc. The ancient Greeks found that square numbers were related to odd numbers. (An odd number is any number that cannot be divided by two.) If you take the sum of any group of consecutive odd numbers, starting with 1, you always get a square number:

$$1 + 3 = 4 = 2 \times 2 = 2^2$$

$$1 + 3 + 5 = 9 = 3 \times 3 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4 \times 4 = 4^2$$

You will notice that the number of odd

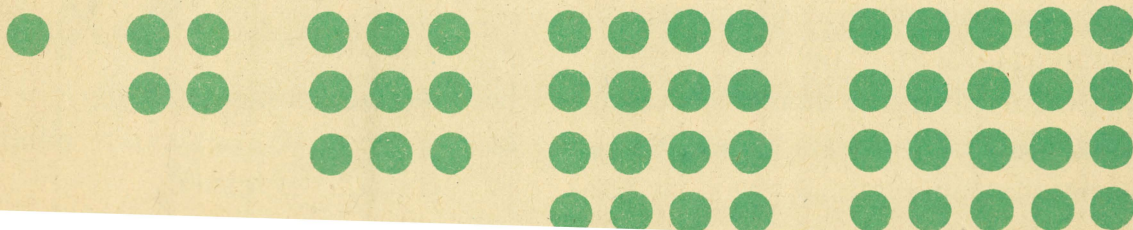


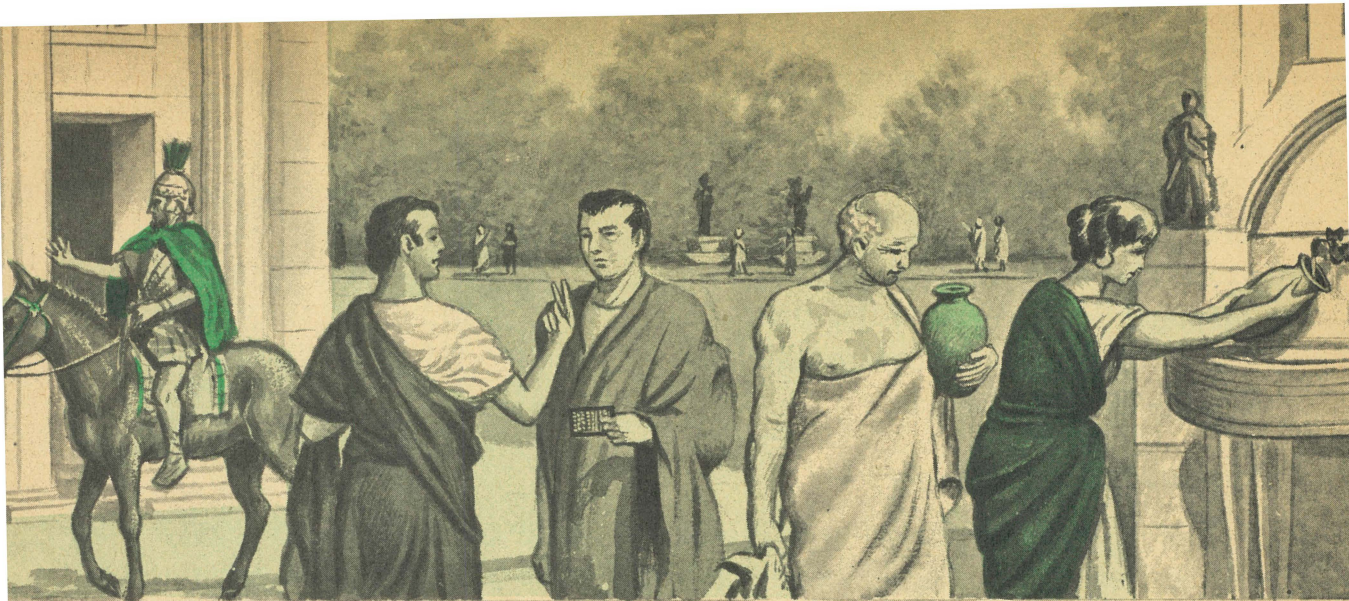
numbers added is always the same as the number to be squared.

What is a perfect number?

To the Greeks, there was mystery about a number which is equal to the sum of all its divisors, except itself. The first such number is 6 . . . $6 = 1 + 2 + 3$. Such a number was called a *perfect number*. The next perfect number is 28 . . . $1 + 2 + 4 + 7 + 14 = 28$. The Greeks discovered the first four perfect numbers: 6, 28, 496 and 8,128.

It was not until some 1,500 years later that the fifth perfect number was found. It is 33,550,336. The sixth perfect number is 8,589,869,056. Up to the present, seventeen perfect numbers have been discovered. The last is so long — it has 1,373 digits — that to write it out would take more than half this page.





From Roman to Arabic Numerals

At the height of their power, when they had conquered most of the then-known world, the Romans were still not able to master the art of simple arithmetic. Those whose work involved mathematics used three methods of calculation: reckoning on the fingers, the abacus, and special tables prepared for this purpose.

In one respect, the Romans had not advanced far from the days of primitive man, for they still used their fingers to count. Finger counting continued in use for hundreds of years after the decline of Rome, as late as A.D. 1100 in Europe.

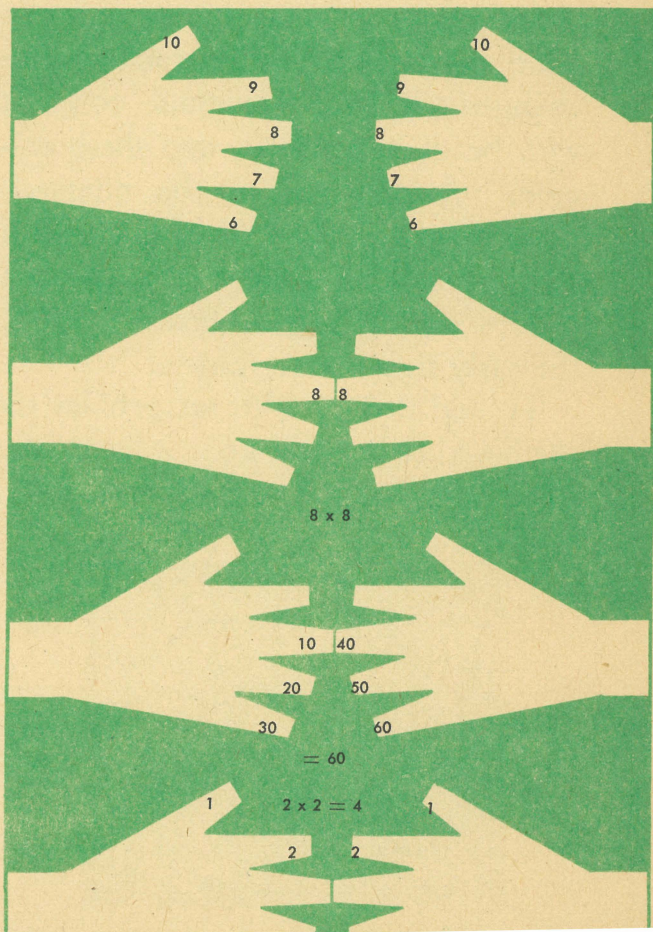
How can you do multiplication with your fingers?

Although finger counting had been used for many centuries before them, the Romans, and even the people of the Middle Ages, could only use this method for addition.

Here is an easy way to check your 6 through 10 multiplication tables by using finger reckoning:

Each finger stands for a number from 6 to 10.

To multiply, put the tips of the two number fingers together. (In the illustration, we are multiplying 8 by 8.)



Count by tens the two fingers that touch and all the fingers below them.

Count by ones on the fingers above those that are touching. Count each hand separately. Multiply the ones of one hand by the ones of the other.

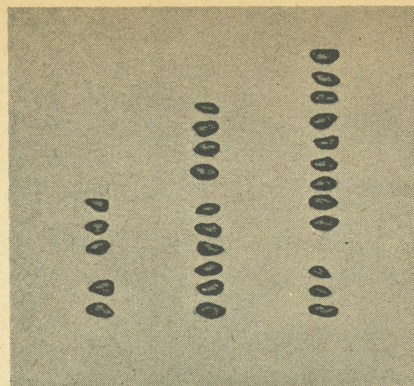
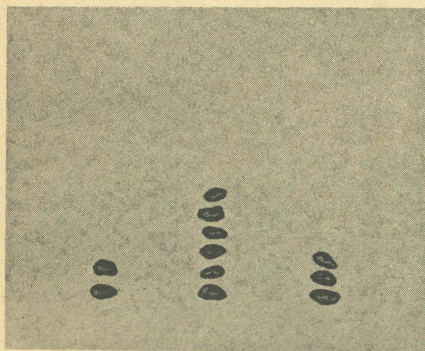
Add the tens number to this last number for your answer.

How does the abacus work?

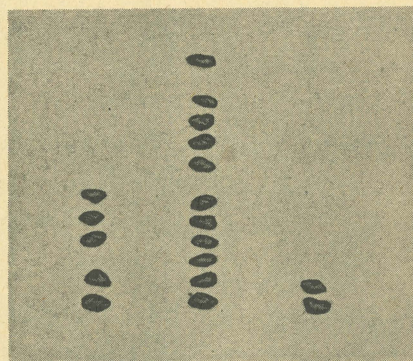
The Egyptians, Babylonians and Greeks used the abacus before the Romans. This simple counting machine was also used by the Chinese and Japanese. Even today, some Chinese and Japanese use the abacus, and they are so expert with it that they can solve problems almost as swiftly as others who use electric calculators.

While the abacus has had many shapes and has had different names, depending upon when and where it was used, its basic operation remains unchanged. It has individual columns with beads or marbles, and these columns are arranged in the numeral-position, or decimal system, of the early Sumerians. The earliest and simplest abacus was a counting board used by the early Babylonian traders.

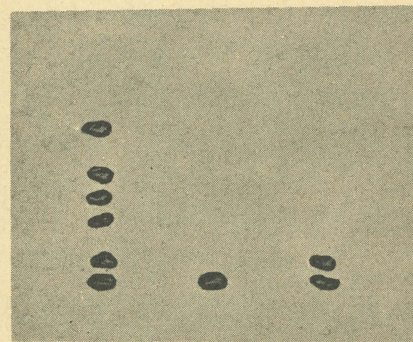
To add 263 to 349, set pebbles on the board to indicate 263: 2 hundreds, 6 tens and 3 units.



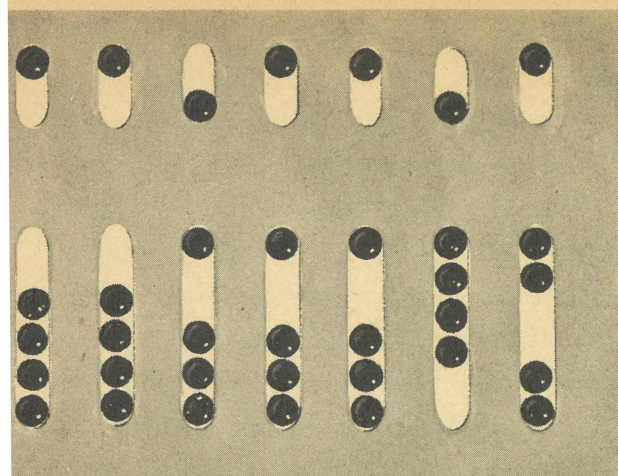
Now add pebbles to signify 349: 3 hundreds, 4 tens and 9 units.



Since no column can have more than 9 pebbles, remove 9 from the units column (on the right), then take another pebble from that column and add it to the tens column.



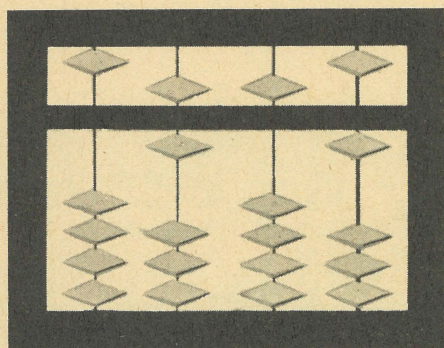
Since there are more than 9 pebbles in the tens column (centre), remove the excess over 9 and add one pebble to the hundreds column (left). The pebbles show the answer: 612.



The Roman abacus was made of metal, and small balls were used in each column. To indicate a number, the balls were placed near the divider. The balls on top equalled 5 and the balls below equalled 1 each.

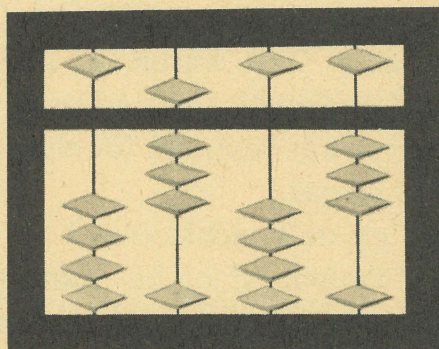
The number shown here is 61,192, using our numeral system.

The abacus in the Orient is called a *suan-pan* by the Chinese and a *soro-ban* by the Japanese. Moving the beads towards the divider of the frame indicates a number. Here is how 651 would be *written*:



There is a 5 (upper bead) and a 1 (lower bead) in the hundreds column, a 5 in the tens column and a 1 in the units column. To add 152 to 651, move 2 unit beads toward the divider in the

unit (right) column. To add 5 to the tens column, move all four lower beads upwards. Since you still need one more to make ten, and since the five or upper bead is already in use, push all the beads in the column away from the divider and add 1 by moving a unit bead toward the divider in the hundreds column. Finally, add 1 bead for the 100 of the 152 you are adding. The beads indicate the answer: 803.



How did the Romans do multiplication?

To do multiplication, the Romans used special tables similar to those used in Egypt and Babylonia. Even with these tables, only highly skilled mathematicians were able to perform multiplication. The Roman numeral system, which is still familiar to us, was difficult to use. Here is an example of how complex even a simple problem could be to the Romans. Let's multiply 18 by 22 using the Roman method:

$$\begin{array}{r}
 \text{XVIII} \quad (18) \\
 \text{XXII} \quad (22) \\
 \hline
 \text{VI} \\
 \text{XXX} \\
 \text{C} \quad \text{LX} \\
 \text{CC} \\
 \hline
 \text{CCC} \quad \text{LX} \quad \text{XXX} \quad \text{VI} \\
 \text{CCCXCVI} \quad (396) \quad \text{ANSWER}
 \end{array}$$

The clumsy number systems used by the Romans retarded the development of mathematics. It was not until centuries after the decline of Rome that a new awakening occurred.

What was the greatest contribution of the Hindus?

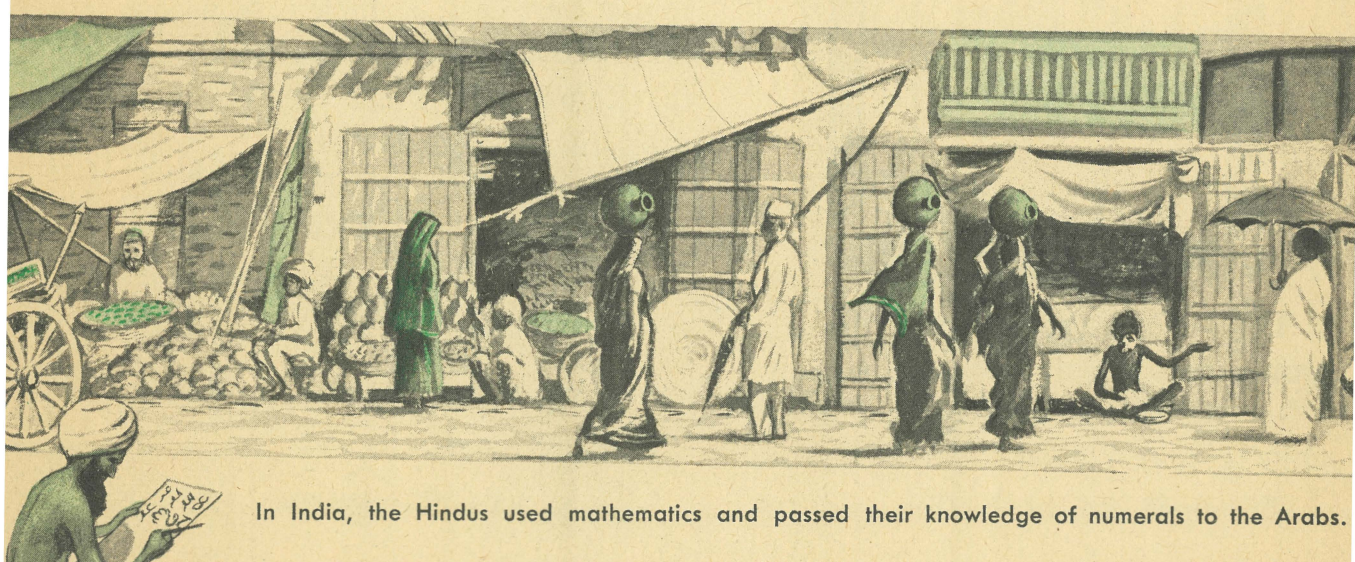
The Hindu civilization, begun in the valley of the Indus River in India, dates back to the early days of Egypt and Mesopotamia. Hindu mathematics was developed to serve astronomy and emphasized arithmetic. As in Egypt, this knowledge was reserved for the select few, the priests.

The greatest achievement of the Hindus, the one which has contributed most to the development of mathematics as a science, was the perfection of our *Arabic numerals*. Although the Babylonians used a *zero* as far back as 2300 B.C., it is the Hindus whom we credit with the origin of the zero in our numeral system. Their earliest symbol was a dot (.) and this later became a

small circle (o) and finally evolved as the zero (0) as we now know it. It is interesting to note that the Hindu word for *zero* means "empty."

The Hindus exhibited great skill in mathematics and did complex problems with very large numbers. The Arabs, who traded with the Hindus, had mastered the Hindu numerical system by the latter part of the eighth century A.D. The Arabs, or Moslems, ruled the Near East, northern Africa and even Spain. It was the Jewish physicians, trained in Moslem schools, as well as the traders, who started the flow of Hindu-Arabic mathematics and numerals into Europe. This knowledge increased greatly during the Crusades, and by the beginning of the fifteenth century, Arabic numerals were widely used by scholars and merchants throughout Europe.

It was this new mathematics — Arabic numerals and the rediscovery of Greek geometry and algebra — that helped Europe leave the Dark Ages and embark on the Age of Discovery.



In India, the Hindus used mathematics and passed their knowledge of numerals to the Arabs.

The head man for the people who
lived and fought in the hills with men
said they're
in the comin'



head for the
hills men
they're
coming

Yotcx sxxxx nxese auvid colho?

Special and important messages are often written in code. The technique of reading coded messages is called *de-coding*. There are many ways in which you can use mathematics to make and solve codes.

Let's take the message: *Yotcx sxxxx nxese auvid colho?* You will notice that this message contains five words or groups of letters and that there are five letters in each group. This is our first clue. Suppose we try to put the first letters of each word together. We would have: *yxnac*. Certainly that does not mean anything. Neither would we find any meaning if we put the second letters of each word together: *osxuo*. What would happen if we wrote these groups backwards? We would have: *canxyouxso*. We have now found our clue!

This illustrates a *square code*. There are 25 letters in the entire message. It is made up of five words of five letters each. That means that the message was

Codes are often used in wartime by military personnel.

written in a square with five columns across and five rows down. We also know that it was written backwards in its coded form.

If we take the coded message and write it backwards to fit the five-by-five square shape, we would find:

c	a	n	x	y
o	u	x	s	o
l	v	e	x	t
h	i	s	x	c
o	d	e	x	x

There is an x used between each word and a double x at the end of the message. We can now read the coded message, which is: *Can you solve this code?*



How can you make a square code?

It is very easy to make a square code. Count the number of letters and spaces between the words. Find the square root, or number which multiplied by itself gives you this total. If the number of letters in the message is 16, then you use the $\sqrt{16}$ which is 4; that is, four columns across and four rows down.

What if the number of letters and spaces is equal to 59? There is no whole number which, when multiplied by itself, equals 59, for $7 \times 7 = 49$ and $8 \times 8 = 64$. Use the next largest number and add several x 's to fill in the blanks anywhere in the message.

All codes are not as simple as this one. Some are very complex and take experts weeks and even months to solve. Here is another one for you to try:

5023355300143500055355100534
3244515000000243401335141500

This one really looks difficult! When you examine it, you find that there are no numbers greater than 5 and that there are many zeros. Does this give you a clue? It wouldn't unless you were a decoding expert. This is a *square number code*. It is made by using a square of 25 boxes:

	1	2	3	4	5
1	a	b	c	d	e
2	f	g	h	i	j
3	k	l	m	n	o
4	p	q	r	s	t
5	u	v	w	x	y

z = 66

Each letter is composed of two numbers. The first number indicates the row across and the second number shows the column down. Thus 23 is the second row and third column, or the letter *h*. In this code, a zero is used between words. You can use any number of zeros you wish, since they do not stand for anything but spaces. The first number of the message is the code signal. In this case, it is 5.

Now decode the message. The 50 indicates that a 5-square was used for the alphabet. The first word, using the numbers up to the zeros, is: 233553. Writing these to form letters, you have: 23 35 53. Using the code, you can find that 23 is *h*, 35 is *o* and 53 is *w*. The word is *how*. You can now solve the rest of the message yourself.

What is a cryptarithm?

The substitution of numbers for letters is called *cryptography*, and a *cryptarithm* is a mathematical problem in which letters are substituted for numbers. Here is a sample:

$$\begin{array}{r}
 ABC \\
 ABC \\
 \hline
 DBC \\
 BCE \\
 ABC \\
 \hline
 ACDBC
 \end{array}$$

The problem is to find the number ABC which has been squared. Here is how to go about solving it:

Start with C which is the last digit of the number and its square. There are only four numbers, which, when multiplied by themselves, will have their last digit the same as the number. They are: 0 ($0 \times 0 = 0$), 1 ($1 \times 1 = 1$), 5 ($5 \times 5 = 25$) and 6 ($6 \times 6 = 36$).

C cannot be equal to 0 , for when you multiply a number by zero, you get zero, but in this problem we multiply C by B and get E .

Neither can C equal 6 . Note in the center column of the addition, we have $D + C + C = D$. If C equals 6 , it would not be possible to add $6 + 6 +$ any number and have the sum equal the missing number.

C cannot be equal to 1 , since $C \times ABC$ would equal ABC , but in this problem it equals DBC . Therefore, C must be equal to 5 .

We also know the number of another letter: A . We see in the multiplication that $A \times ABC = ABC$. Therefore, A must equal 1 .

We now have two digits: $A = 1$ and $C = 5$. Write the problem over substituting the known numbers:

$$\begin{array}{r}
 1B5 \\
 1B5 \\
 \hline
 DB5 \\
 B5E \\
 1B5 \\
 \hline
 15DB5
 \end{array}$$

Look at the centre column where $D + 5 + 5$ is used in the problem. We now know that $10 + D$ equals D and we carry 1 . Therefore, we know that $1 + B + B = 5$. The only number that B can stand for is 2 since $1 + 2 + 2 = 5$.

The problem is now solved: $ABC = 125$.

Here is another cryptarithm for you to solve. The answer will be found below, upside down.

$$\begin{array}{r}
 DEF \\
 DEF \\
 \hline
 FGF \\
 DEFE \\
 \hline
 DDEGF
 \end{array}$$

Answer to cryptarithm:

$$\begin{array}{r}
 DDEGF = 11025 \\
 D = 1 \quad E = 0 \quad F = 5 \quad DEF = 105
 \end{array}$$

What is a magic square?

Magic squares have intrigued people for more than 2,500 years. Actually, there is no magic in the square, for it is only an addition table in which the numbers are arranged in a tricky way. Draw a square divided into nine boxes as shown in the illustration.



Now, put in the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, using each number only once and putting only one number in each box. The numbers have to be arranged so that no matter which way you add them — down a column, across a row or along either diagonal (from one corner to the opposite corner) — the sum is the same.

We know that the sum of all the numbers from 1 through 9 is 45. If you divide this by 3, you will get 15. Therefore, each set of three numbers has to equal 15. Try to solve the magic square before you read the solution below.

How can you make a magic square?

Here is the solution to the magic square problem. You can use this method to make a magic square of any size so long as you have an odd number of columns and rows.

Begin by writing the number 1 in the centre box in the top row. Lift your pencil lightly and move diagonally to the right and up. You are now outside the square. Therefore, continue down the next column to the last row. Write the number 2 in that box. Again move diagonally upward and to the right, and again, you are outside the square. Move across the row above to the last box in the square and write the number 3.

8	1	6
3	5	7
4	9	2

The first step in constructing a magic square is shown in red. The second step is in grey. The third is in white.

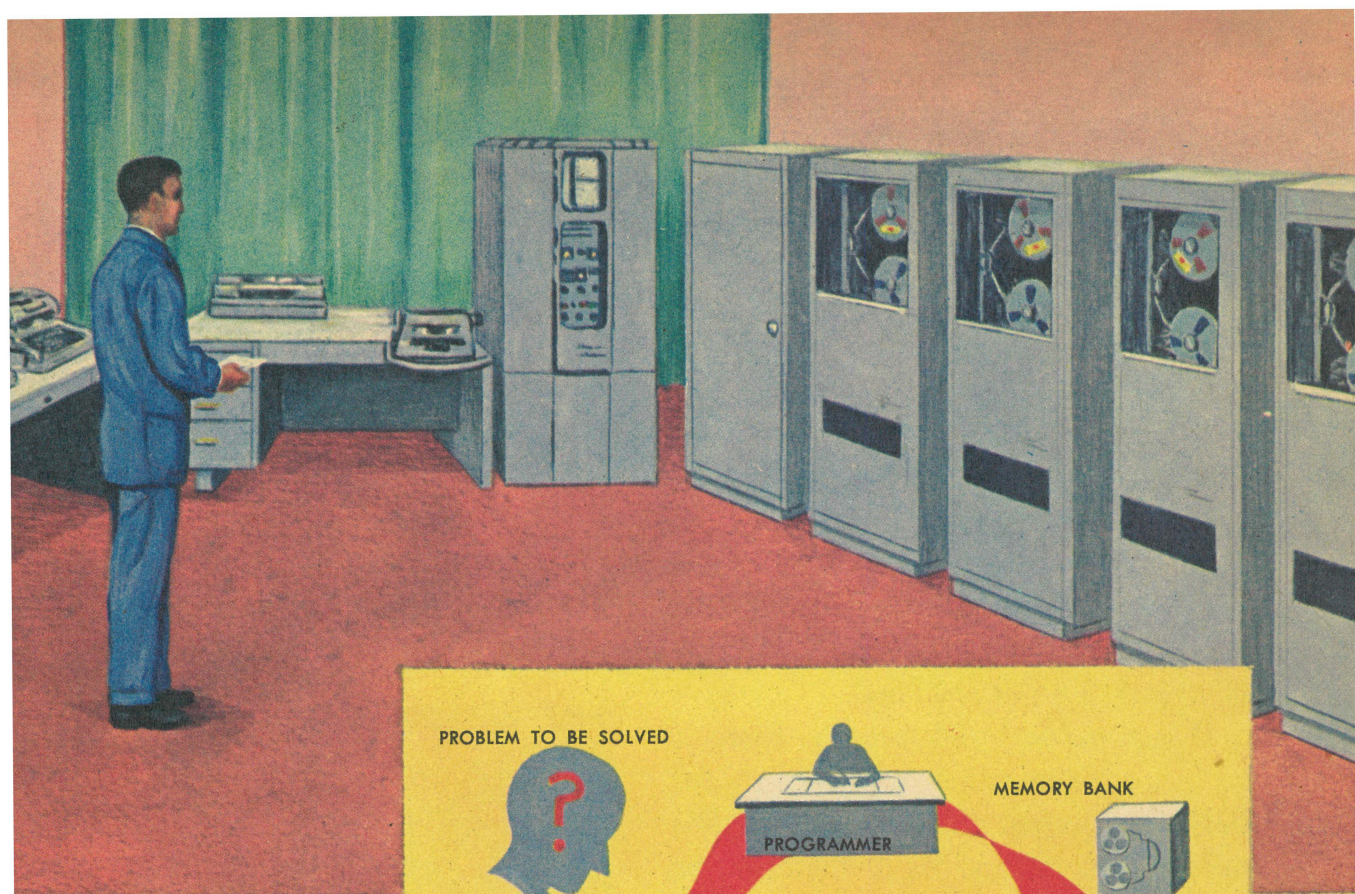
You have now completed the first set of numbers. A set of numbers is equal to the number of boxes in any column or row — in this case it is three. To begin the next set, move down one box and write the first number of the set, 4. Repeat the same technique used with the first set. Move one box up and one box to the right. Now write the number 5 in that box. Again, move one box up and one to the right. Enter the number 6. You have now completed the second set. To start the third set, go down one box and write the number 7 and then complete the set.

How can you solve the price tag mystery?

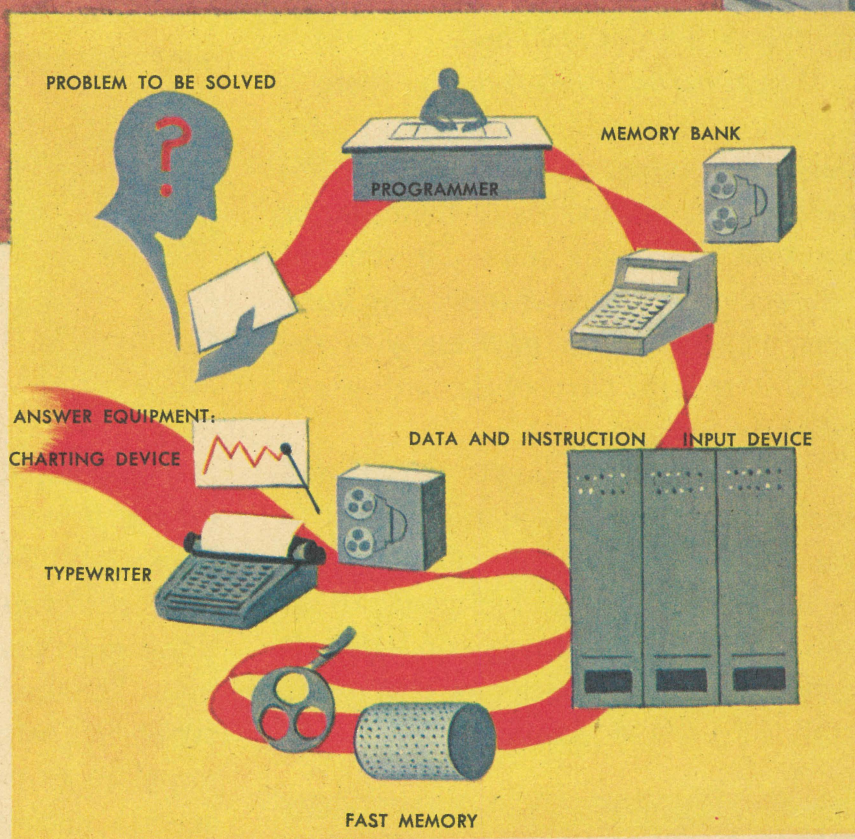
Have you ever made a purchase and found cryptic markings on the tag or box? Usually, they are letters, a secret code, which tell the storekeeper the price or age of the product. There is really no mystery as each letter stands for a number. The merchant often selects a name or a word and assigns a letter to each number.

H a r o l d S m i t h
1 2 3 4 5 6 7 8 9 0 .

With this code, *Hl/Ht* is translated into 15s. 10d. This may be the cost of the object to the storekeeper and he knows that he has to sell that item for more money in order to make a profit. On the other hand, a package may be coded: arhdhdH—which translated means 23.6.61 or 23rd June 1961. This tells the merchant when he bought the product. Note that capital H decodes as 1; a small h decodes as a full stop. This method of writing code is not new. Both the ancient Greeks and Hebrews used letters to write numbers



A modern computer is a complex set of electronic equipment. In scene (top, left to right) are special typewriters to translate problem into computer language on cards and tape; programmer translates problem and feeds instruction in with problem; "fast memory" unit of the computer where information is stored; "memory bank" from which "fast memory" picks up data. Finally, the electronic circuit sets out the answer on a typewriter on the programmer's desk.



Calculating With Computers

In minutes, sometimes even in seconds, electronic computers solve problems which would take mathematicians weeks or months to do. Computers today can guide rockets to land on any spot on the earth or to go into orbit

around the earth, moon or sun. Computers also help to design and test new airplanes and later guide them through the sky. These modern machines can forecast weather, check the income-tax returns of millions of people, compute

payrolls for thousands of workers, and do countless other jobs in only a fraction of the time required by men.

What do *digital* and *analog* mean?

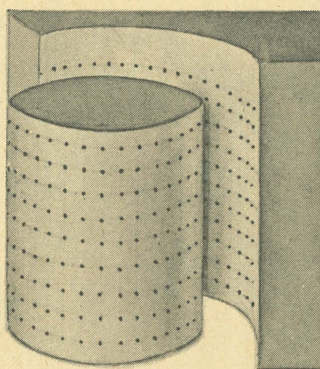
Basically, there are two types of computers: *analog* and *digital*. *Digital* refers to the familiar Arabic numerals which stand for definite quantities. Finding that the sum of $6 + 7 = 13$, is a digital computation. The digits themselves have a meaning, but the units (whether they be chairs, rockets or miles) are not specified.

When we deal with *analog*, there is a basic unit to which we always refer back, like yards, miles or gallons. We say we bought 3 yards of material, or we walked $\frac{1}{2}$ of a mile. The unit of reference is the yard or mile. The numbers 3 and $\frac{1}{2}$ show how many of the units — or what parts of the unit — are being considered.

There are many simple digital and analog computers which are familiar to you. The office adding machine is a digital computer. The fuel gauge of an automobile is an analog computer. It indicates what proportion or part of the tank is filled with gasoline.

How does the computer work?

In both the digital and analog computers, the operator punches the instructions (or *commands* as they are called in computer language) and the numbers of the problems on special tape or cards. When the tape or cards are placed in the machine and the power is turned on, the holes in the cards or tape allow tiny electric pulses to travel through and mark the magnetized drum or fast memory of the machine.



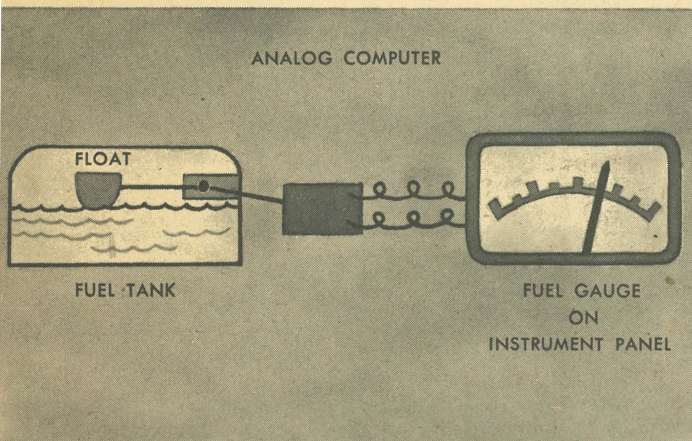
The computer's magnetized drum, known as "fast memory."

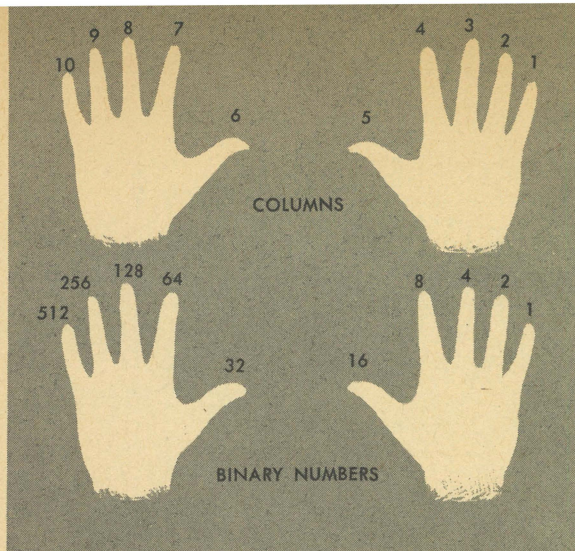
The computer takes the information on the drum, draws upon its memory banks, and interprets all the data. It then performs the necessary calculations and puts the results on magnetic tape. The tape is either fed to a special typewriter, which translates the electric impulses into language, or operates a special machine, which draws a chart or graph.

What is the *binary* system?

There are two different mathematical systems used by computers. Some work with the numbers we use all the time, the *decimal* or *ten-base* system. Others use the *binary*, or *two-base* system. Although the binary numbers look

ANALOG COMPUTER





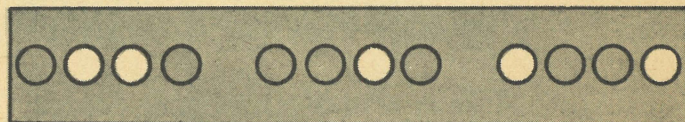
The position or column in the binary system has a specific value. For example, 100 in binary notation means that the 1 in the third column equals 4; the 0's stand for nothing. Thus, the 100 binary notation = the number 4. Here's another example: 10110 in binary means that the fifth column equals 16, the 1 in the third column equals 4 and the 1 in the second column equals 2. That is $16 + 4 + 2$ or 22. Therefore, the binary number, 10110 equals 22.

strange, they are formed in exactly the same way as the decimal system.

The binary computer works on what engineers call a *flip-flop circuit*. It is somewhat similar to the light switch in your room. There are two positions: *on* and *off*. Symbols or numerals are used for these two positions: 1 for *on* and 0 for *off*. If you have a series of

OUR NUMBER	PANEL LIGHTS			
0	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
4	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
6	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
7	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
8	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>

Here is how our numbers (zero to ten) would look on a single panel binary computer which has four lights.



Here is a problem: If you saw three panels of binary lights as shown above, would you know what number the machine was showing? The answer is 629.

switches to turn the panel lights *on* and *off*, you can use different combinations to form various numbers.

Mathematics of the Space Age

What are the space problems of mathematics?

There are many problems in space-age exploration. Some are being solved by psychologists, chemists, physicists and biochemists, but many of the problems are in the hands of mathematicians. Three of the problems which engage the mathematician are:

Computing the varying resistance of the air on a rocket.

Calculating the effects of the earth's rotation on the direction of a rocket or satellite.

Determining initial air speed and momentum force required to counter the earth's gravitational pull.

A rocket speeding through the air of

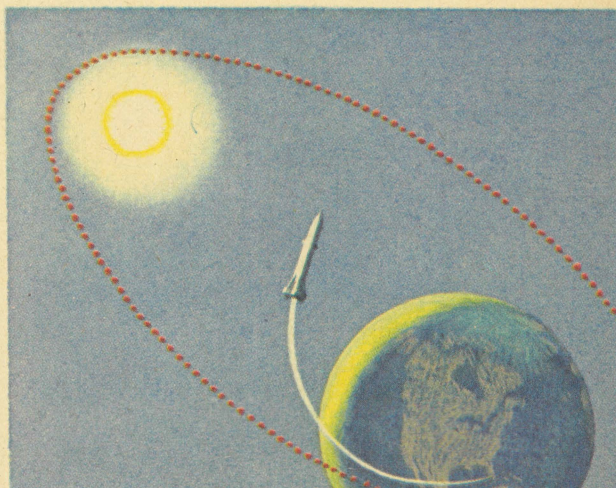


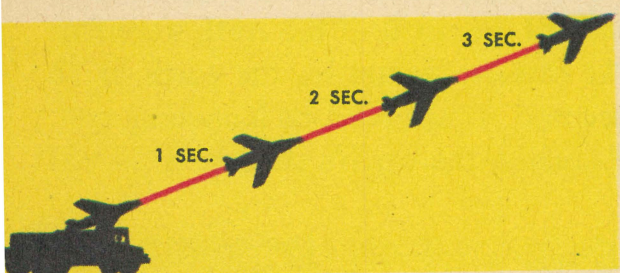
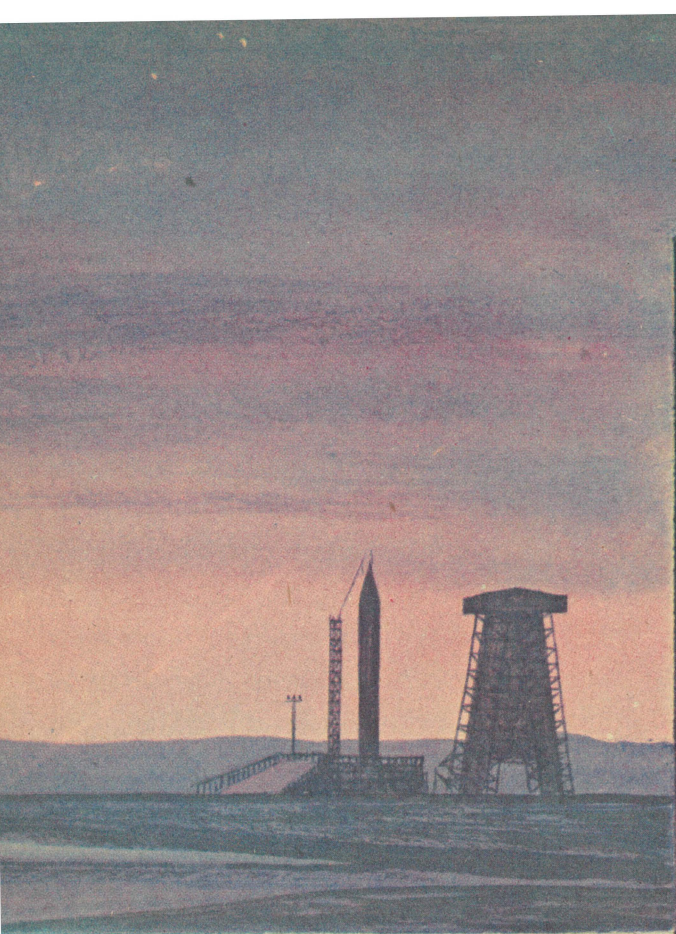
our atmosphere faces resistance forces similar to those met by a submarine plowing through the depths of the ocean. The faster the rocket goes, the greater the resistance of the air. It has long been known that this resistance increases with the square of the velocity. This means that as the speed is doubled, the air resistance is multiplied by four. It is now known that once the rocket exceeds nine miles per minute, the air resistance increases with the cube of the velocity. In other words, every time the speed doubles after the rocket has reached nine miles a minute, air resistance is multiplied by eight.

If the earth were standing still when a rocket was fired, it would be a simple problem to compute the path of the rocket. However, the earth is constantly in motion. Not only is it revolving on

its axis, but it is also moving in an elliptical orbit around the sun. It is necessary to compute the path of the rocket in three dimensions, taking into account the earth's movements, air resistance and the earth's gravitational pull.

With the help of computers, mathematicians carefully plan the path of flight and the increases in speed.



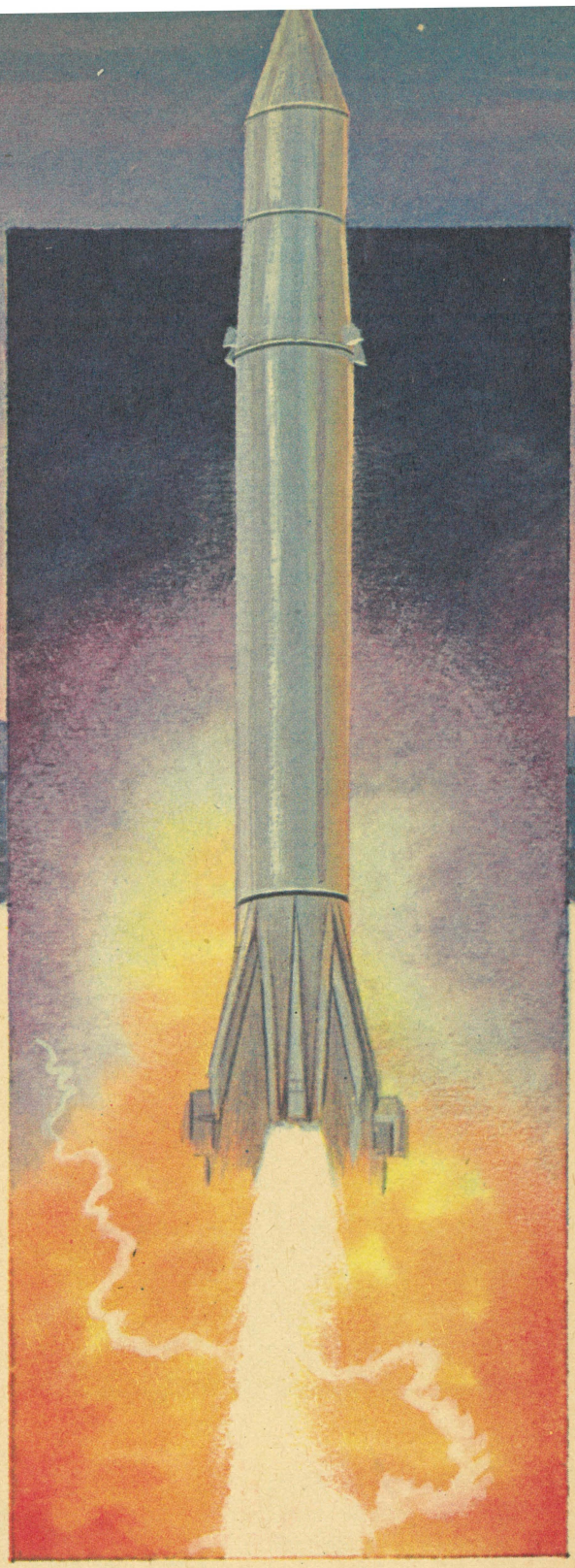


Without gravity, this would be the path of a rocket — a straight line upward from the loading-firing frame.

How does gravity affect a rocket's flight?

We do not know exactly what gravity is, and Albert Einstein, in his *Theory of Relativity*, does not use *gravity* at all. If there were no gravity, we would be able to fire a rocket into the air and its path would be a straight line.

Yet, we know that whatever is thrown into the air or dropped, will come down. We also know that the time for an object to strike the ground is the same whether it is dropped or

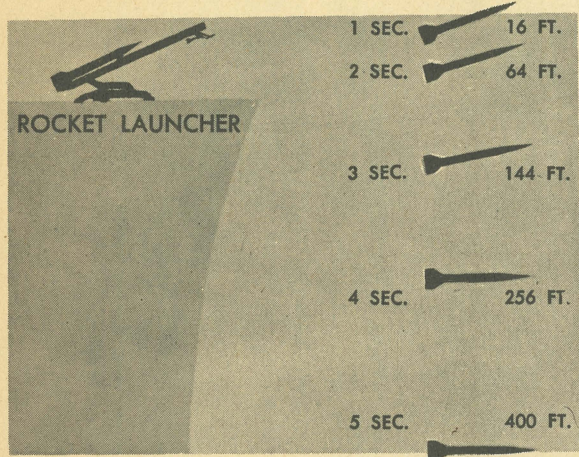


Launching a rocket requires precision mathematics.

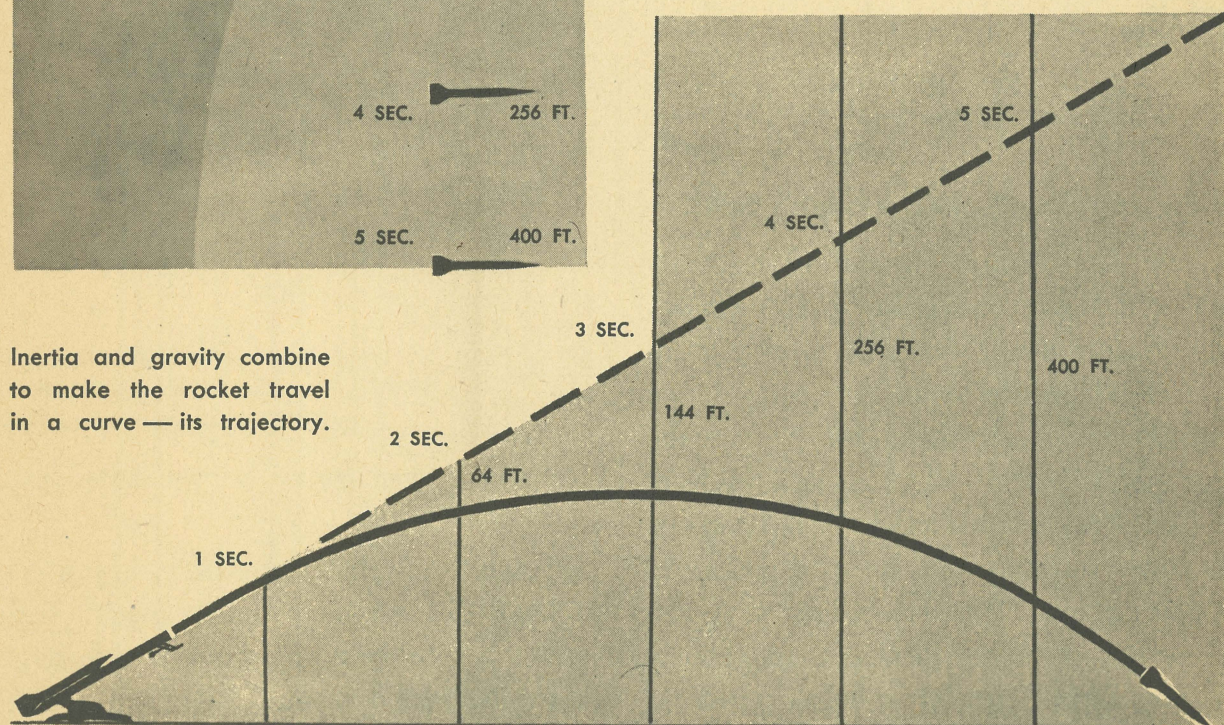
thrown straight out. Isaac Newton found the precise mathematical formula to measure the force which acts on a falling body, and called it gravity.

The formula is $s = \frac{1}{2} gt^2$, where s is the distance an object falls toward the earth, t is the time, and g is the acceleration or increase in speed due to gravity, which is 32 feet per second per each second it falls.

Apply this formula to a rocket fired from a launching pad on top of a very high mountain. The diagram shows how the rocket would fall, due to gravity, if it had no forward speed. It drops 16 feet in the first second, 64 feet in two seconds, 144 feet in three seconds, etc. But the rocket also has a forward speed or initial thrust. Therefore, as it moves forward in its original direction, the pull of gravity acts upon the rocket so that it travels in a curve and finally comes back to earth. The curve made by a body in space is called a *trajectory*.



Inertia and gravity combine to make the rocket travel in a curve — its trajectory.

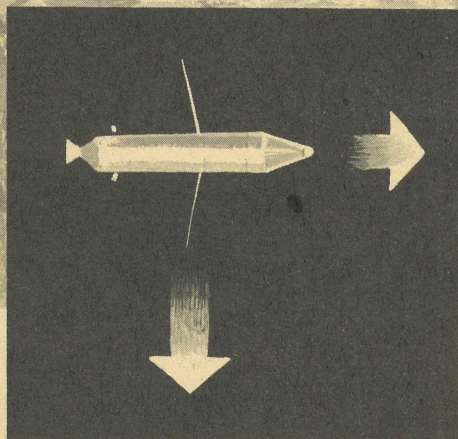
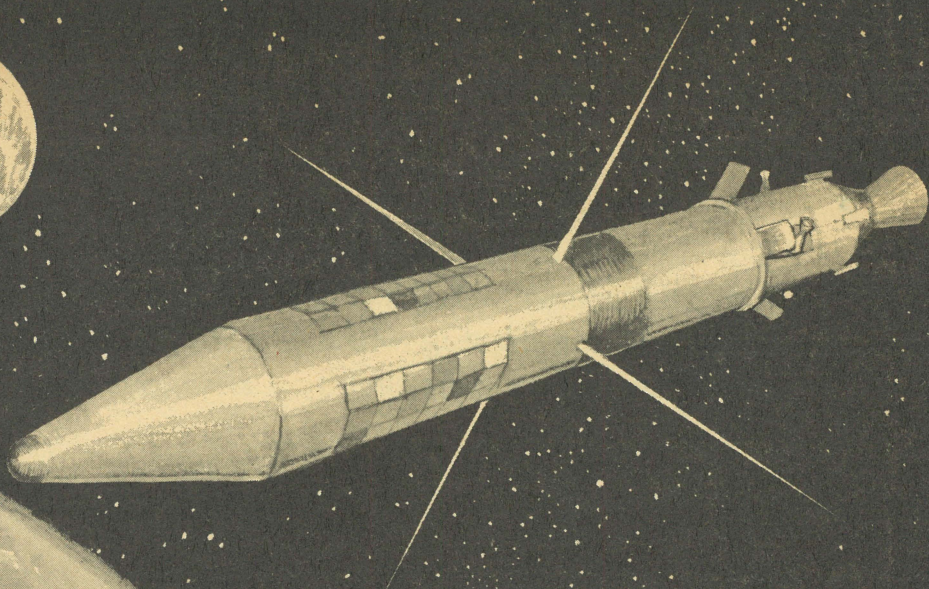


What keeps a satellite in the sky?

The basic principle of gravity also affects the rocket that will launch a satellite and affects the satellite, too. Using Newton's gravity formula, an object falls 1 mile in 18 seconds. This information helps us to keep the satellite in the sky.

Let us skip the involved mathematics necessary to get the satellite up into the air one mile above the earth's surface. As soon as it gets there, the satellite is affected by two forces. One is its own speed to carry it forward. The other is the earth's gravity pulling it down.

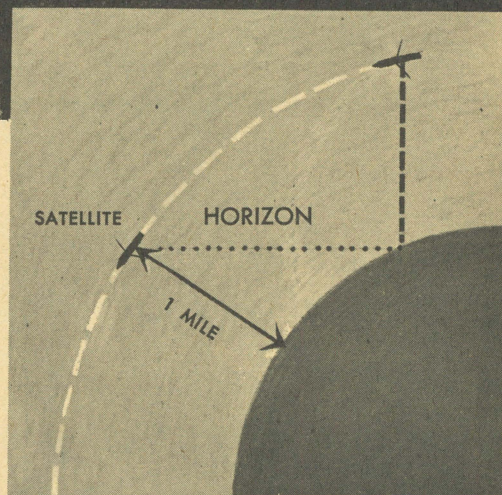
To make the calculation simple, suppose the forward speed of the satellite is 1 mile per second. In the first 18 seconds, the satellite will travel 18 miles along the horizon and will fall 1 mile toward the earth. If the earth were flat, the satellite would hit the earth in 18 seconds, 1 mile from its launching pad. Actually, the earth's surface is curved



As soon as a satellite is in orbit (top), it is affected by two forces: *inertia* and *gravity*. The balance of these forces creates the satellite's path.

and it would take the satellite a fraction of a second longer than 1 second before it hits the earth.

The problem is to determine how fast the satellite must go so that it orbits around the earth. We know that it will fall 1 mile in 18 seconds. Its speed must be so great that it would remain 1 mile above the earth's curved surface. At this point, it would travel 90 miles in a horizontal direction in 18 seconds and reach a point where the earth's surface is 1 mile below. This speed is equal to 18,000 miles per hour.



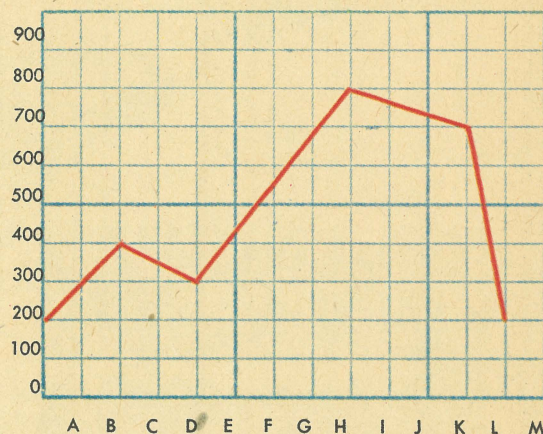
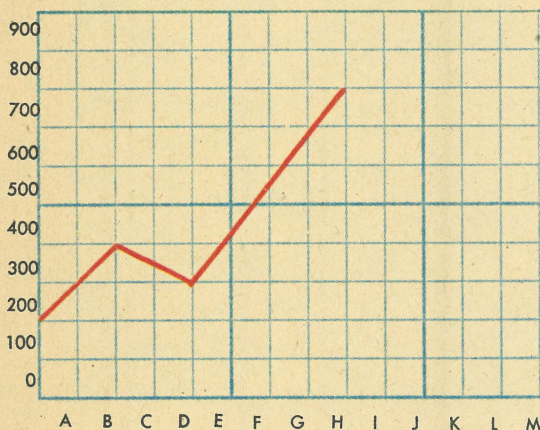
Speed needed to orbit satellite: 18,000 mph.

Even at that speed, however, it would not really be possible to keep the satellite in motion, since we have not considered air resistance. In addition, we have to shoot our satellites to heights of 100 miles or more so that they do not burn themselves up in the earth's atmosphere. At this height above the earth, the air is thinner — there is less friction and less heat.

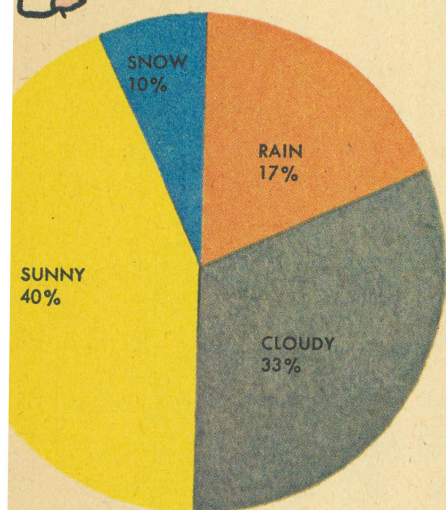
What Is a Graph?

Rene Descartes, a mathematician who lived in France at the time that the Pilgrims came to America, was the first to use a *graph*. It is a drawing which tells its story through lines, bars, circles

and figures. Graphs can show negative and positive numbers, like profits and losses, or degrees above and below zero. Statisticians use graphs to convey a message in its simplest form.

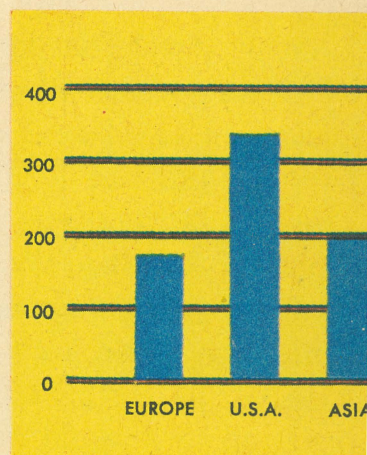
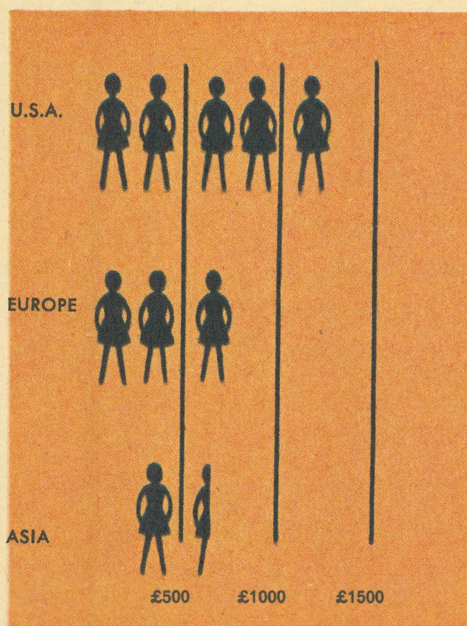


"How's business?" These line graphs tell the answer.



A pie graph shows parts of a whole.

A pictograph gives basic information by using pictures.



A bar graph is a comparison of two, three or more items.

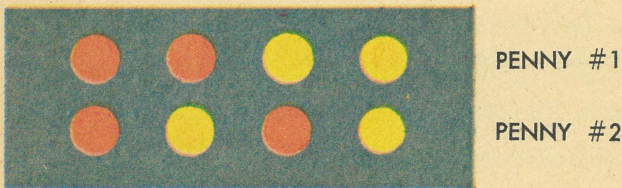
What Are Your Chances?

What is probability?

When you toss a coin into the air, it will land either *heads* or *tails*. If you toss a penny, what are your chances that it will fall *heads* up? Suppose you tossed two pennies at one time. What are the chances of both falling *heads* up? Are your chances the same in both cases?

If you flip a single penny, it will fall either *heads* or *tails*. The mathematician would say that the *probability* of the penny landing *heads* up is $\frac{1}{2}$ or *one-of-two*. Or, you have one chance out of two that it will fall *heads*.

On the other hand, if you flip two pennies at the same time, there are four ways in which they can land. They can fall with two heads, one head and one tail, one tail and one head, or two tails, as shown in the illustration.



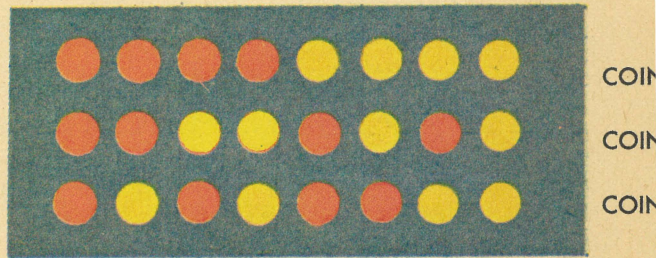
Orange stands for "heads"; yellow stands for "tails."

There is one chance of four that any one of these combinations will occur. The probability would be $\frac{1}{4}$, or one-of-four, for two heads, $\frac{1}{4}$, or one-of-four, for two tails, and $\frac{2}{4}$, or $\frac{1}{2}$, or one-of-two, for a head and a tail.

How good is your hunch?

When you guess that the coins will fall with both heads up, you are usually

playing a hunch. There is no reasoning behind your guess. But statisticians put probability to work to take the guessing out of hunches. What would happen if you tossed three coins into the air at the same time. You would find the following possible combinations:



Orange stands for "heads"; yellow stands for "tails."

There are eight possible combinations. The chances of getting all three coins to fall with their heads up is $\frac{1}{8}$, or one-of-eight. This is the same for getting all three tails. But the chances of two heads and one tail is $\frac{3}{8}$, and the chances of two tails and one head is also $\frac{3}{8}$.

Were you to continue tossing an increasing number of coins, you would be able to determine the chances for any possible combination.

What is Pascal's triangle?

Mathematicians have figured out the chances for an almost unlimited number of combinations. These chances are usually presented in a triangular form, known as *Pascal's triangle*. To learn your chances, refer to the proper line of the triangle. For example, if you are to play two games of tennis against a

				1		1																		
				1		2		1																
				1		3		3		1														
				1		4		6		4		1												
				1		5		10		10		5		1										
				1		6		15		20		15		6		1								
				1		7		21		35		35		21		7		1						
				1		8		28		56		70		56		28		8		1				
				1		9		36		84		126		126		84		36		9		1		
				1		10		45		120		210		252		210		120		45		10		1

player who is just as good as you are, what are your chances of winning both games? Look at line two (from the top) of the triangle. According to the *laws*

of chance as stated in Pascal's triangle:

You have $\frac{1}{4}$, or one-of-four, chances to win both games.

You have $\frac{2}{4}$, or $\frac{1}{2}$, or one-of-two, to win one and lose the other.

Finally, you have $\frac{1}{4}$, or one-of-four, chances to lose both games.

If you want the possibilities for eight games, refer to line 8 and you will find that you have $\frac{1}{256}$, or one-of-two hundred fifty-six, chances to win all eight games.

There is a very interesting number pattern in the Pascal triangle. Each number is the sum of the two numbers immediately above it.

New Mathematics

The fastest growing and most radically changing of all the sciences today is mathematics. It is the only science in which most of the theories of 2,000 years ago are still valid and in which there is still room for new ideas and new branches of mathematics.

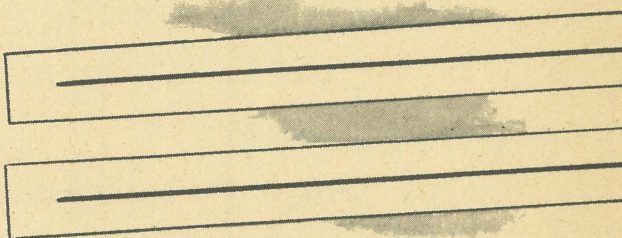
What is topology?

One of the most active branches of mathematics today is *topology*, a form of geometry. It is a departure from the Euclidean geometry of rigid lengths and angles and shapes that never change. Topology does not consider size but only shape, and the shape can be folded, stretched, shrunk, bent, distorted in almost any way, but never

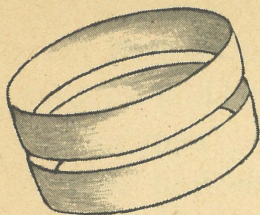
torn. It has been said that since all solids pierced by a hole look alike to a topologist, he is a mathematician who cannot tell the difference between an automobile tyre and a doughnut.

What is a Moebius strip?

Here is a sample of simple topology. Cut two pieces of paper about 1 inch wide and 10 to 12 inches long. Draw a line down the centre of each strip. Take

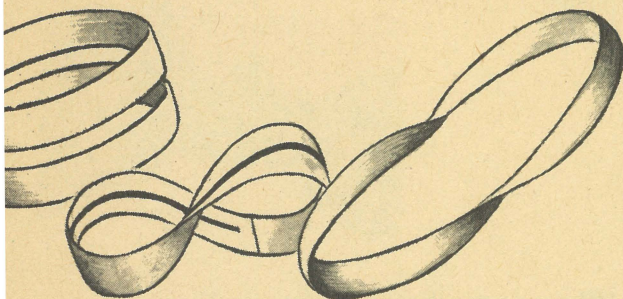


one of the strips of paper and join the ends together with Sellotape or paste.



Use a pair of scissors and cut along the centre line. You have two rings.

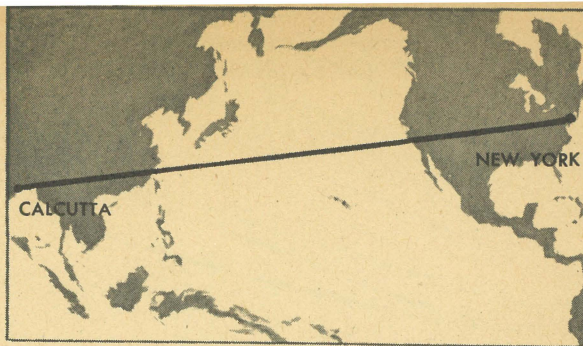
Now take the other strip. Give it a half-twist and join the ends. Take a pair of scissors and cut along the centre line.



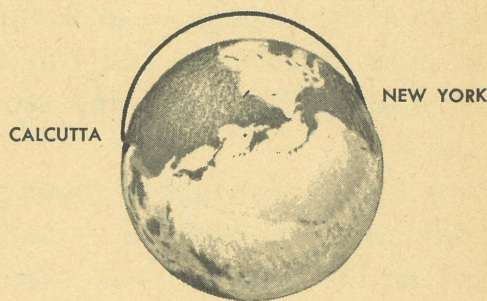
What do you have? One large circle! This is a *Moebius strip*, named after the German astronomer of the early nineteenth century, Augustus Ferdinand Moebius, who first investigated the curious properties of topology.

When is a straight line not a straight line?

Mathematicians define a straight line as the *shortest distance between two points*. If you looked at a flat map of the world and used a straight line to find the shortest distance between New York and Calcutta, India, you would find that the path goes across the Atlantic Ocean to Morocco, across Africa, the Arabian Sea and India.



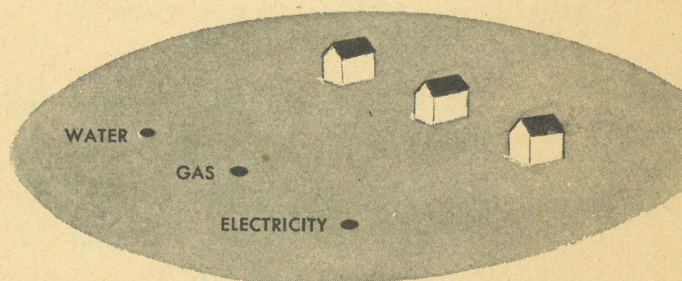
But the world is not flat. Instead, if you find the shortest distance on the surface of a globe, the path goes northward over Canada, Greenland, the Arctic Ocean above the top of Europe,

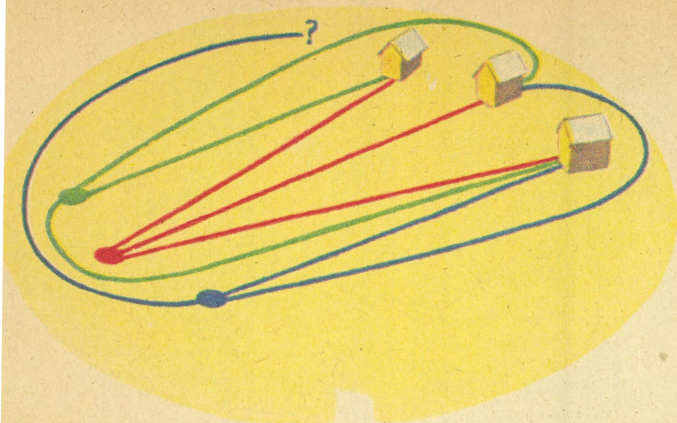


over Soviet Siberia, Tibet and Nepal into India. This is called the *Great Circle route* and it is, indeed, a strange straight line, since it curves with the curvature of the earth.

How do you do the impossible?

Here is a problem which cannot be solved with ordinary mathematics. There are three houses near each other. Connect each of the houses with the water, gas and electricity supplies so that no lines cross each other.

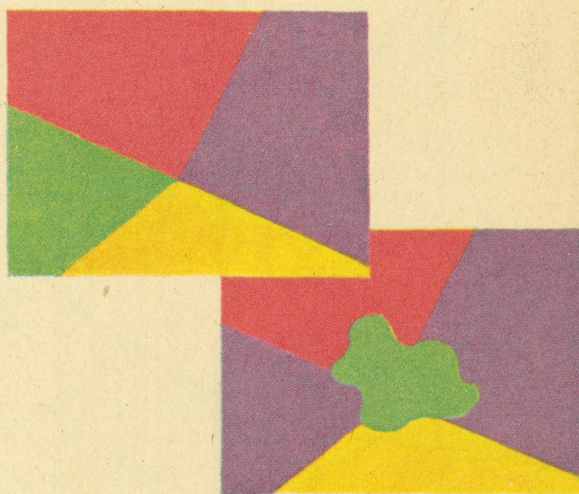




With the Euclidean geometry of flat surfaces, we could arrive at a situation as shown in the diagram — all but one of the lines connected. You can try other possibilities yourself and see if you can do it.

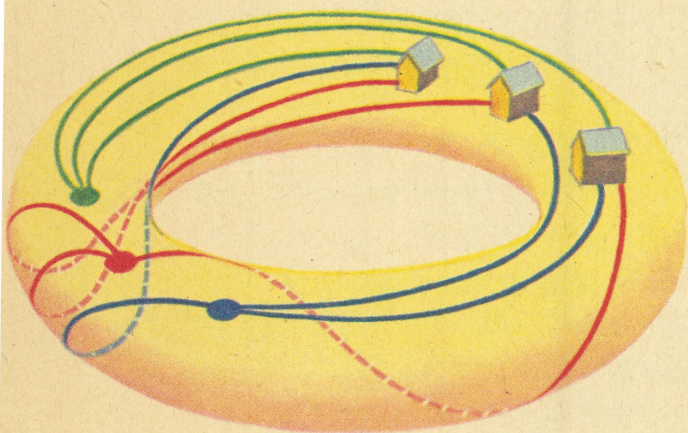
Using the mathematics of topology, the solution to this problem — as described in the diagram — is exceedingly simple. We use a *torus* or doughnut surface, instead of a flat or plane surface.

This answer involves topological mathematics. It has been found from experience that no matter how complicated the map, no matter how many countries it contains, and how the countries are situated, the map can be coloured by using only four separate colours. However, no mathematician thus far has been able to produce the mathematical proof that this is true.



Where is mathematics headed?

Mathematics is an essential part of the cultural heritage of our world. New rules are being formulated and new fields — non-Euclidean geometry, algebraic topology, linear programming, matrix algebra, game theory probability — are being explored and developed. In this space age, changes are taking place not only in practical, applied mathematics but also in theoretical mathematics. There is still much ahead in this special field of science. The need for further development, the challenge, the fascination, are all there. Perhaps some day, one of you will make an important contribution to the world of mathematics.



How many colours do you need to colour a map?

In colouring maps, it is customary to use different colours for any two countries that have a common boundary. What is the minimum number of colours a mapmaker needs?